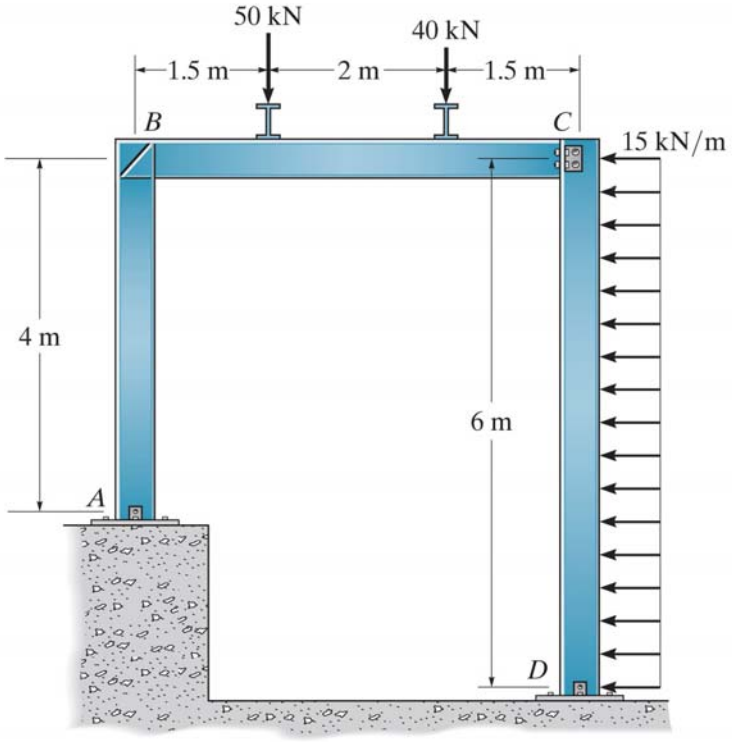
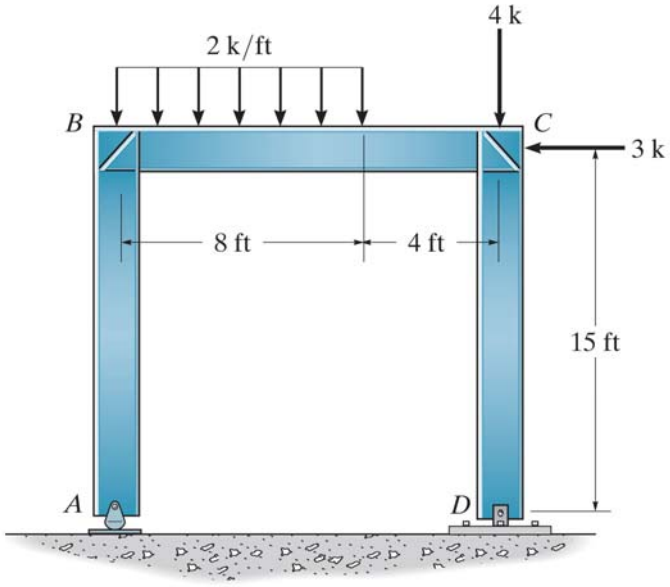
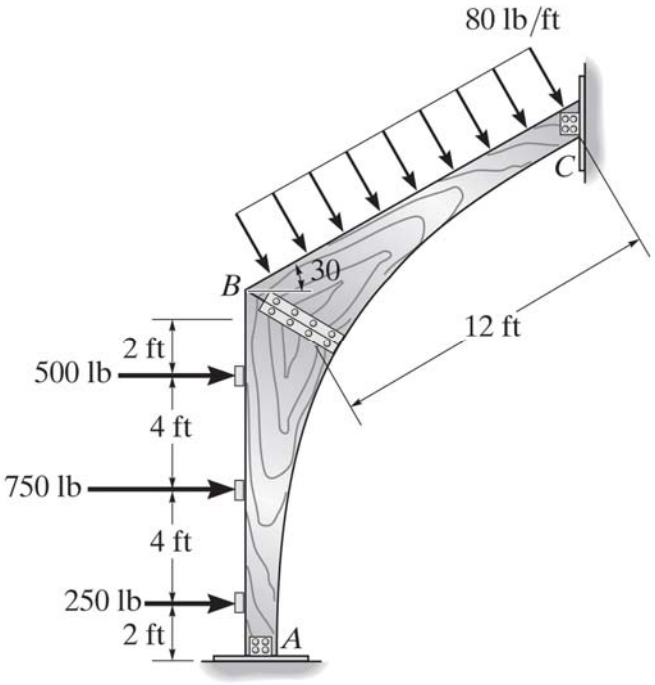
(1)


50 kN
40 kN
1.5 m
2 m
1.5 m
B
C
15 kN/m
4 m
6 m
A
D

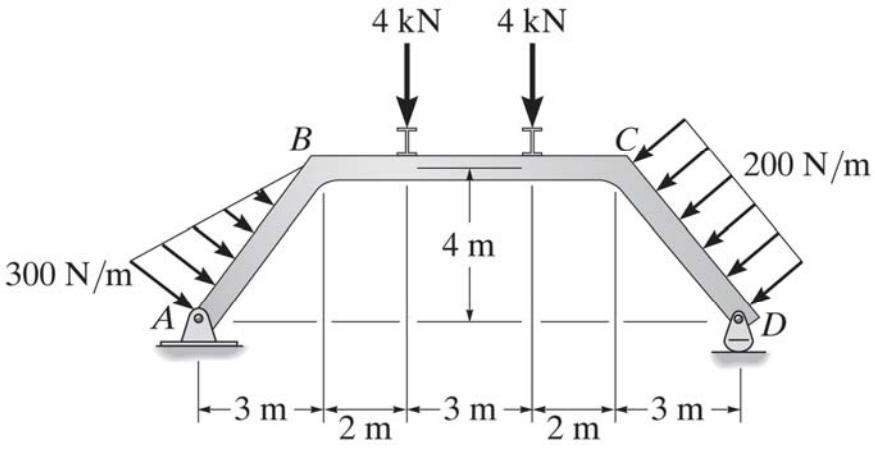
(2)


4 k
2 k/ft
B
C
3 k
8 ft
4 ft
15 ft
A
D

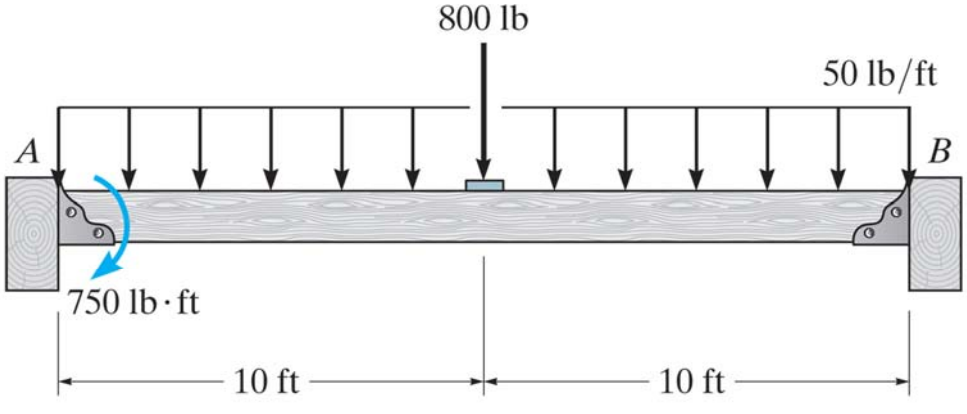
(3)

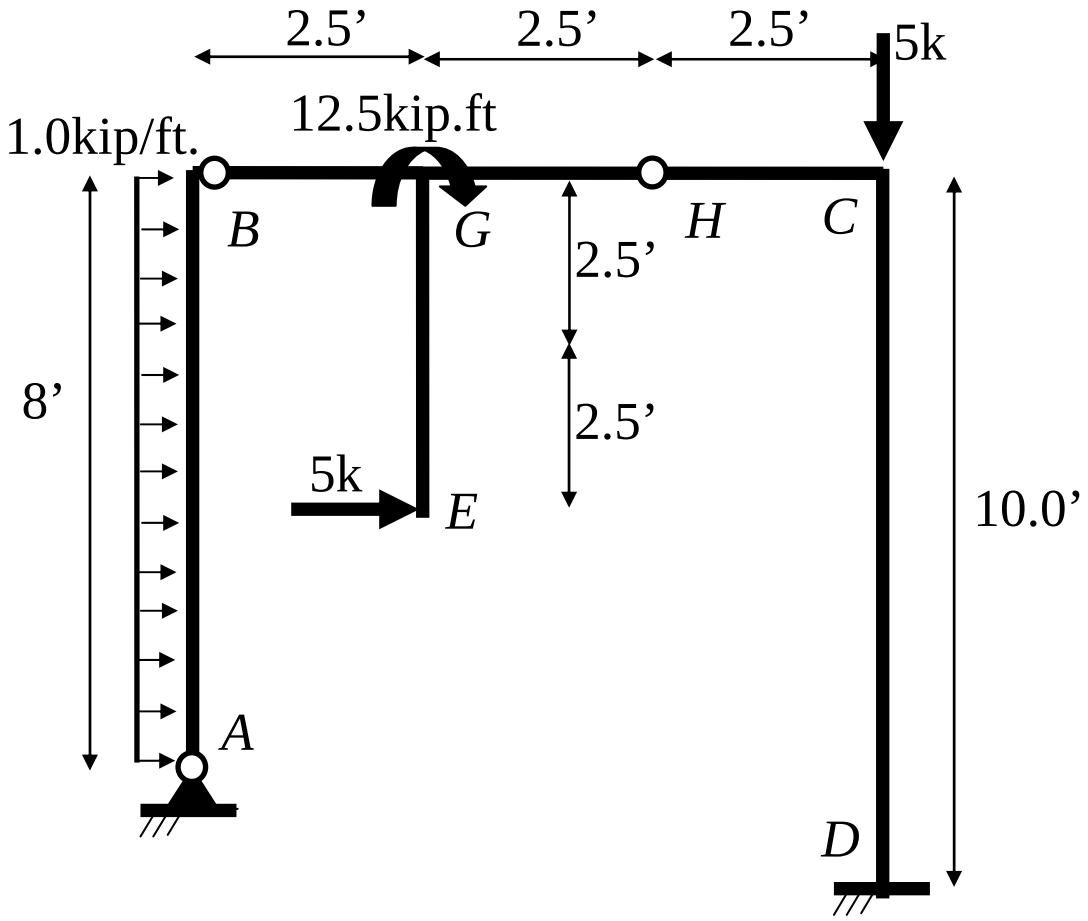

80 lb/ft
C
B
30
12 ft
2 ft
500 lb
4 ft
750 lb
4 ft
250 lb
2 ft
A

(4)

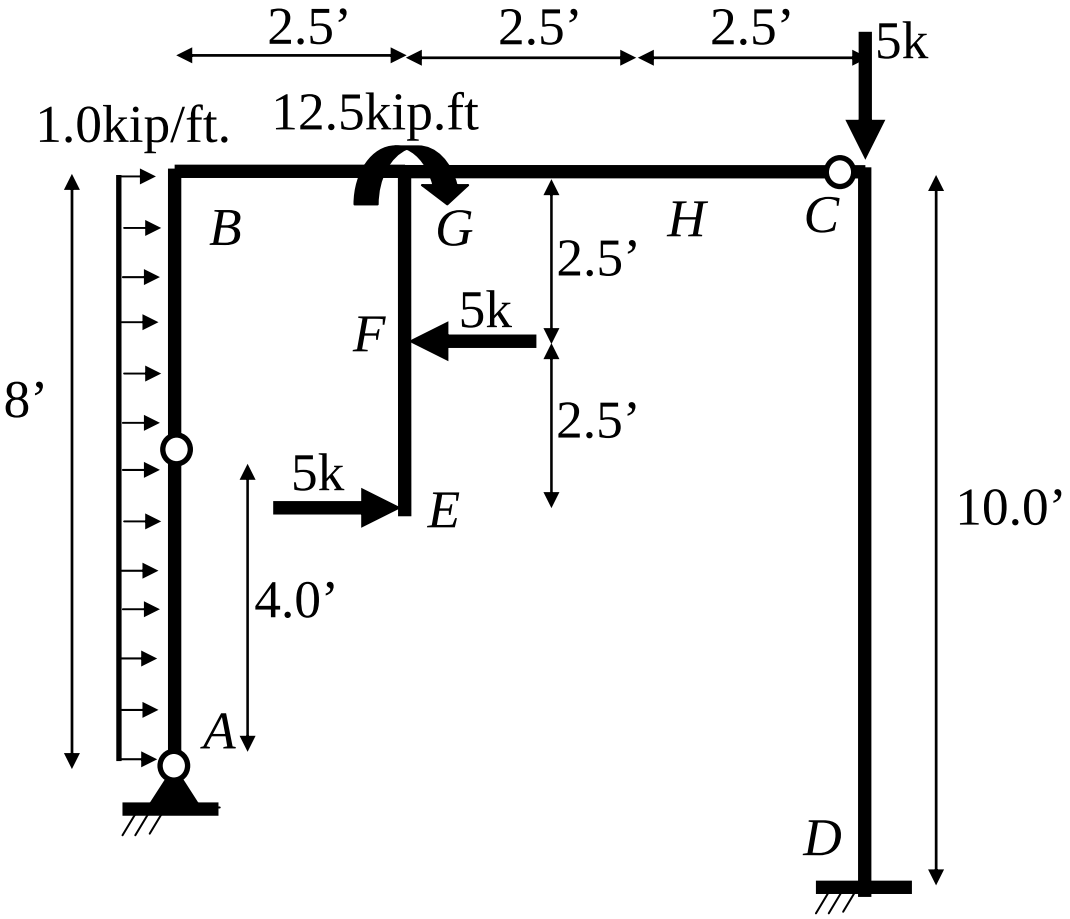

4 kN
4 kN
B
C
200 N/m
4 m
300 N/m
A
D
3 m
2 m
3 m
2 m
3 m

(5)

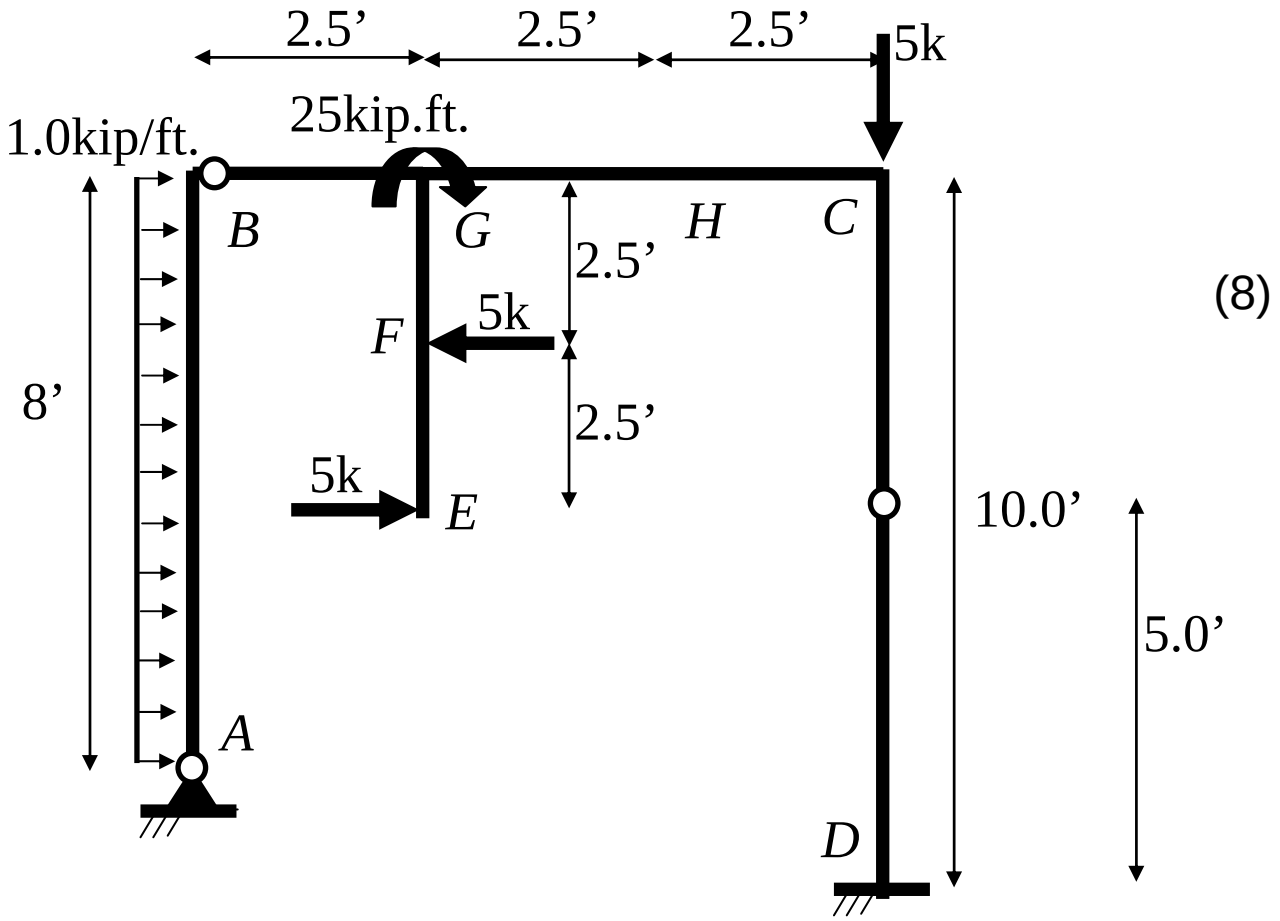

800 lb
50 lb/ft
A
B
750 lb · ft
10 ft
10 ft



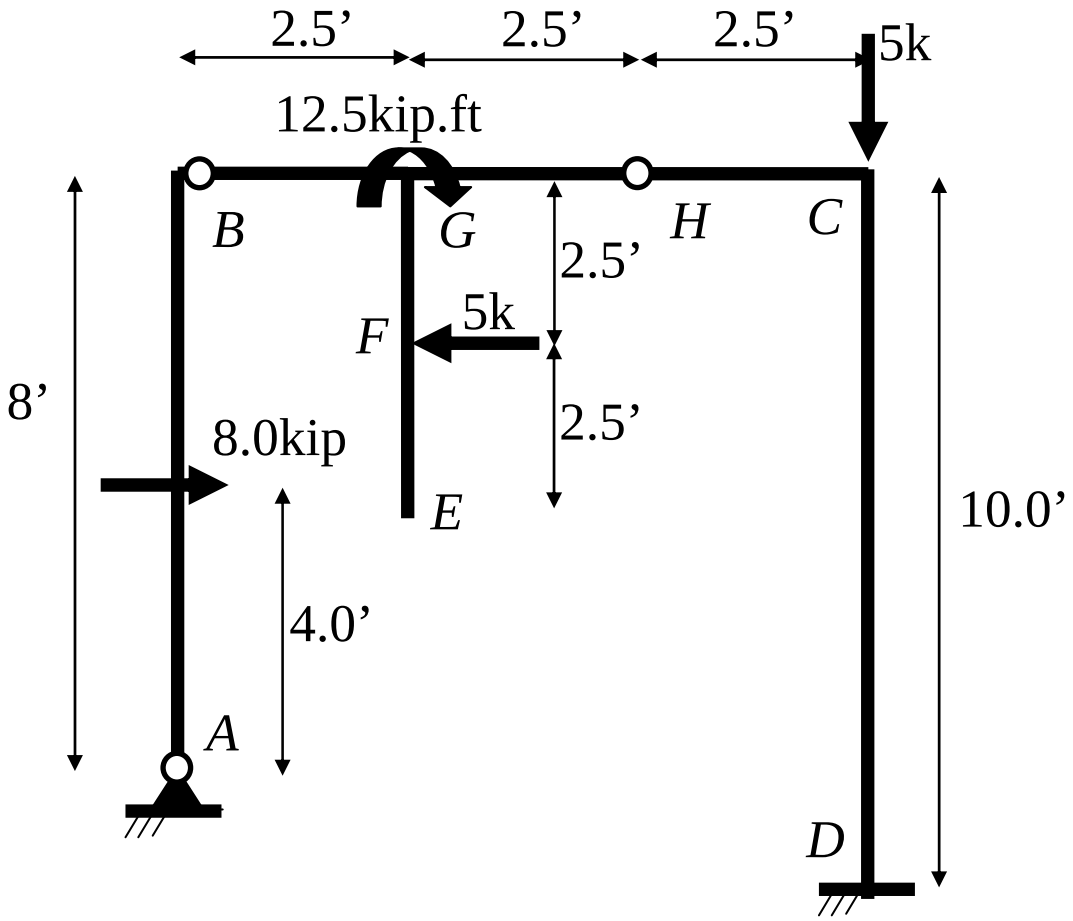
(6)



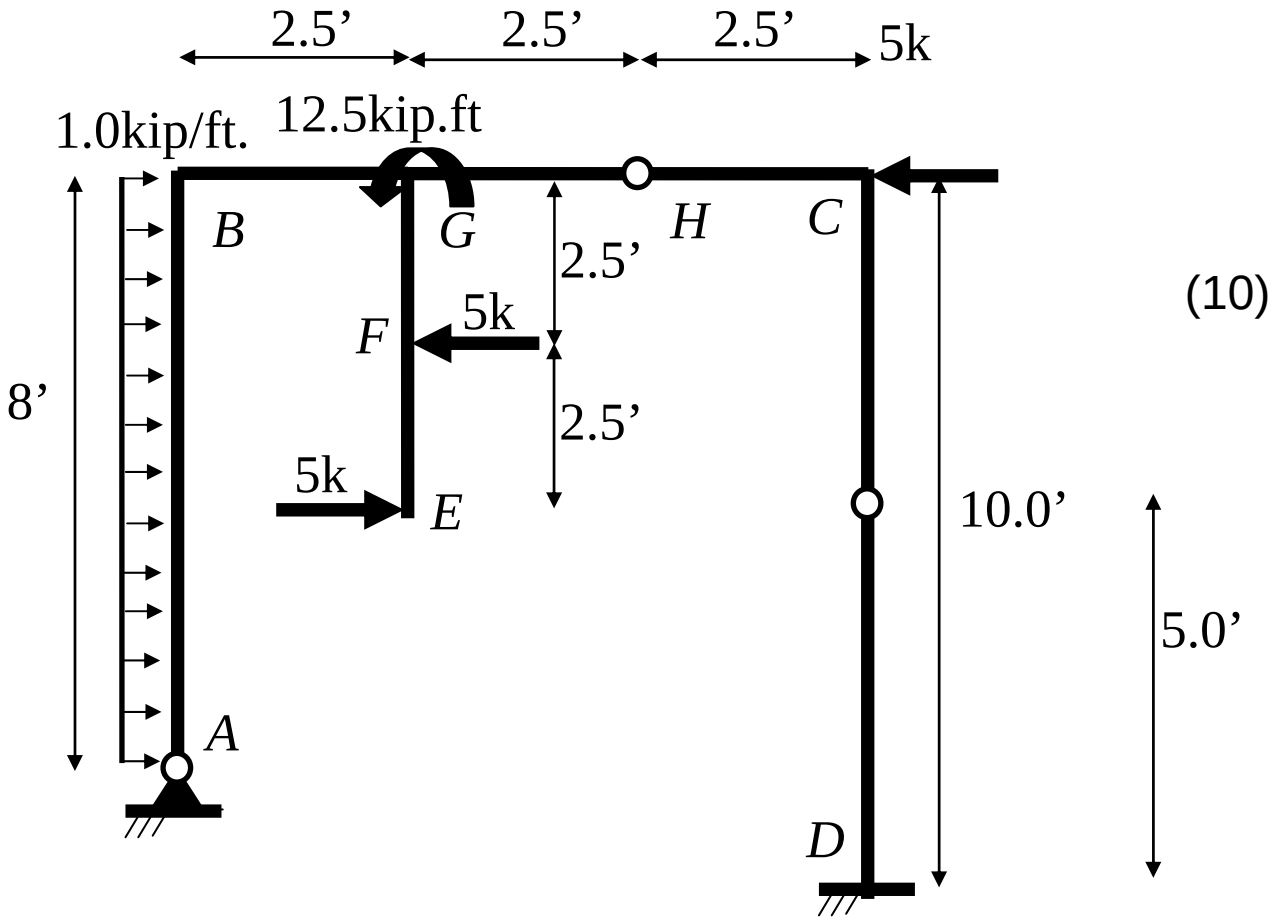
(7)



(8)

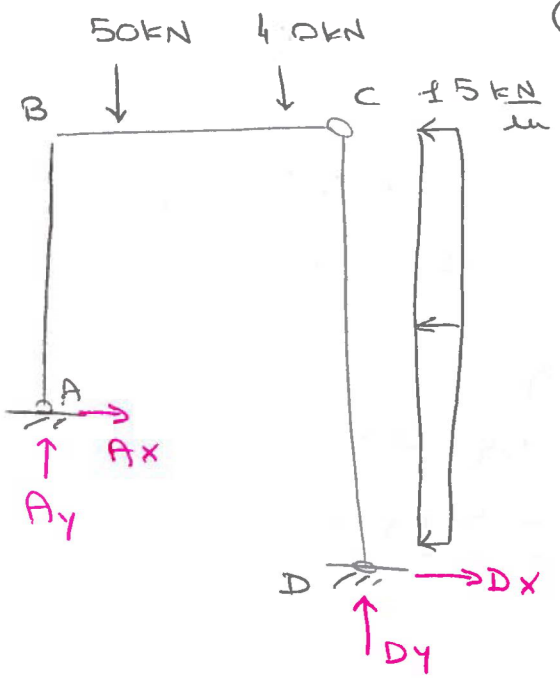


(9)



(10)

①



① $\Sigma M_C (II) = 0$

$15 \times 6 \times 3 - Dx \cdot 6 = 0$

$Dx = 45\ kN$

② $\Sigma Fx = 0$

$Ax + Dx - 15 \times 6 = 0$

$Ax = 45\ kN$

③ $\Sigma M_C (I) = 0$

$Ay \times 5 - Ax \cdot 4 - 40 \times 1.5 - 50 \times 3.5 =$

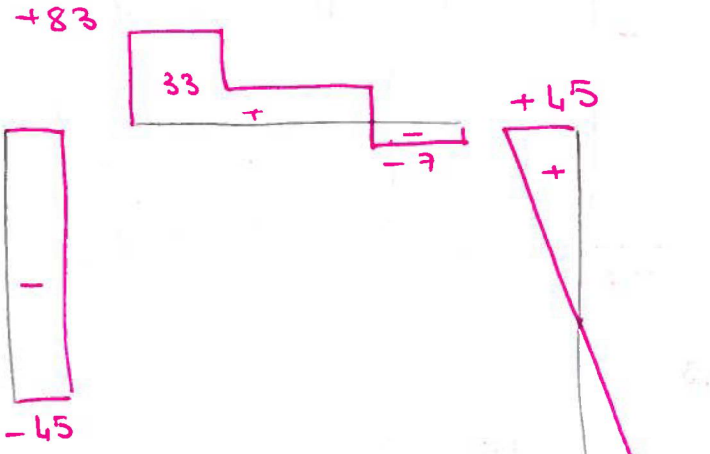
$Ay = \dfrac{45 \times 4 + 235}{5} = 83\ kN$

④ $\Sigma Fy = 0$

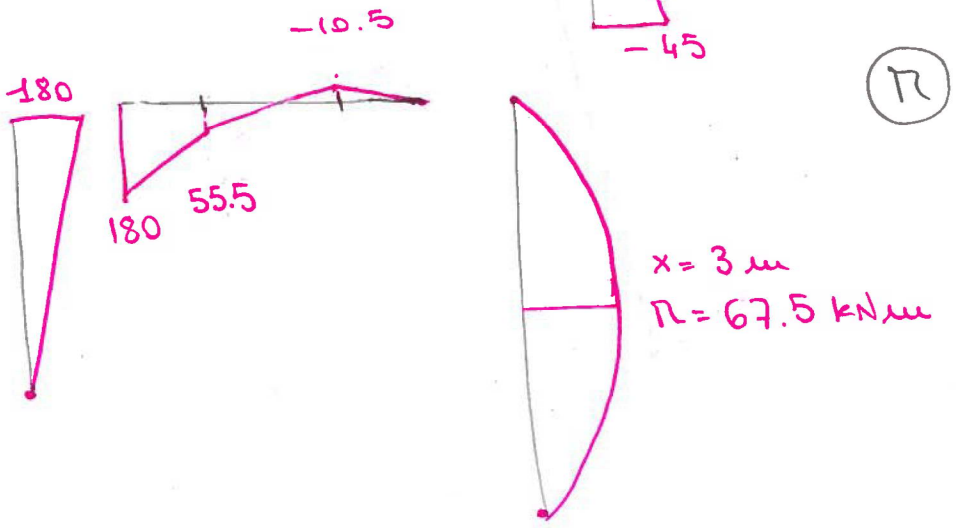
$Ay + Dy - 50 - 40 = 0$

$Dy = 90 - Ay = 7\ kN$

V



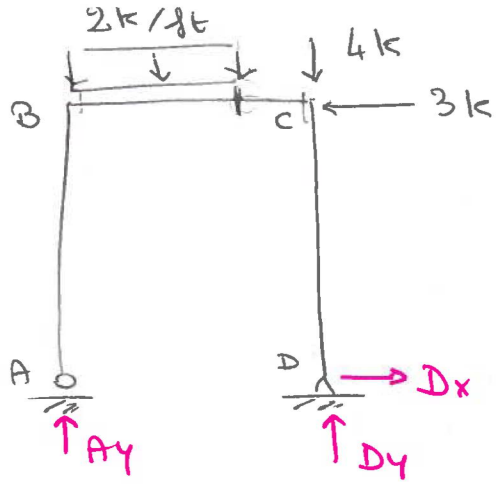
M



$x = 3\ m$

$M = 67.5\ kNm$

(2)



(1) $\Sigma F_x = 0$

$D_x = 3k$

(2) $\Sigma M_D = 0$

$-3 \times 15 - 2 \times 8 \times 8 + A_y \cdot 12 = 0$

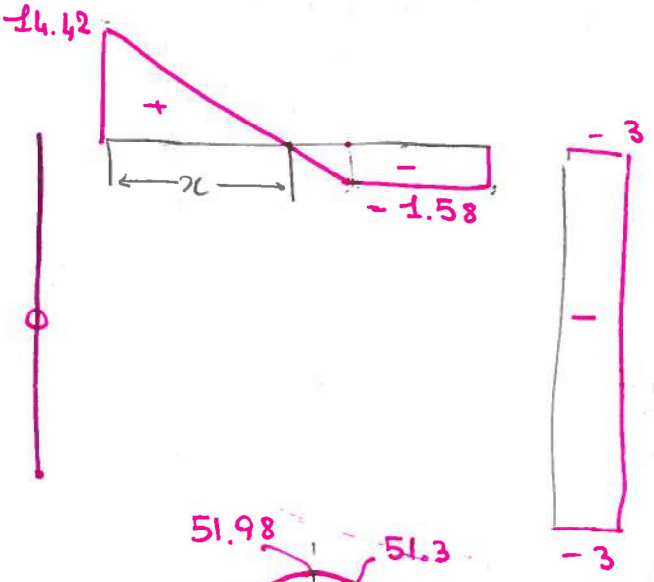
$A_y = 14.42\,k$

(3) $\Sigma F_y = 0$

$A_y + D_y - 2 \times 8 - 4 = 0$

$D_y = 5.58\,k$

(V)



$\frac{14.42}{16} = \frac{x}{8}$

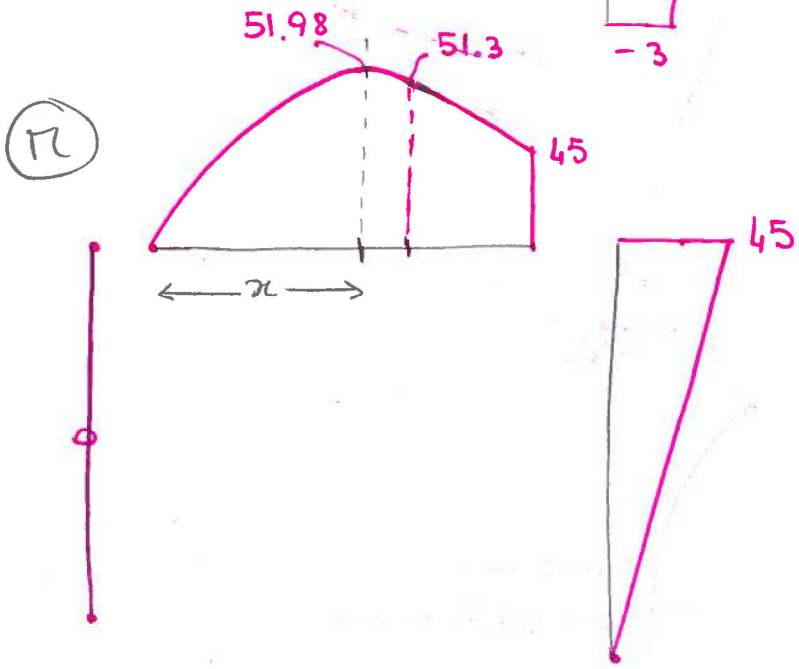
$x = 7.21\,ft$

$M_x = M_B + \frac{V_B \cdot x}{2} =$

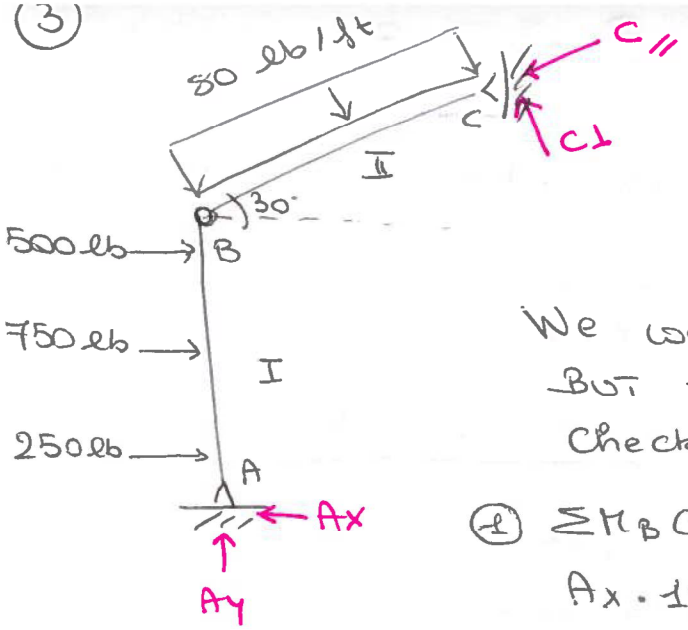
$= 0 + \frac{14.42 \cdot 7.21}{2} =$

$= 51.98\,k\,ft$

(M)



(3)



NOTE: We don't need the value of Ay & C// to draw shear & moment diagrams!

We won't find these reactions, BUT this means that we can't check our results.

(1) $\Sigma M_B(I) = 0$

$Ax \cdot 12 - 500 \times 2 - 750 \times 6 - 250 \times 10 = 0$

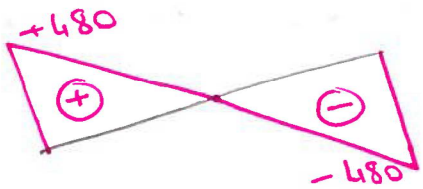
$Ax = 666.67$ lb

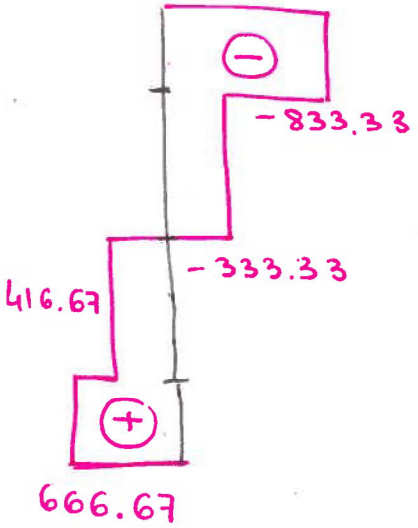
(2) $\Sigma M_B(II) = 0$

$80 \times 12 \times \frac{12}{2} - C_1 \times 12 = 0$

$C_1 = 480$ lb

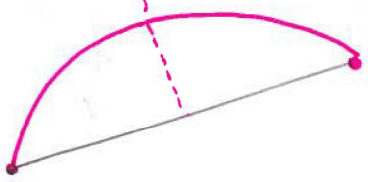
(V)

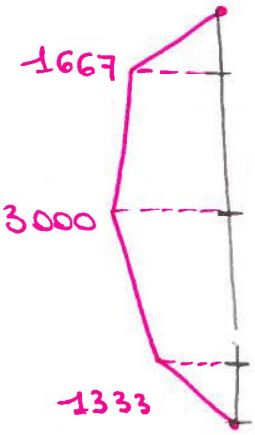


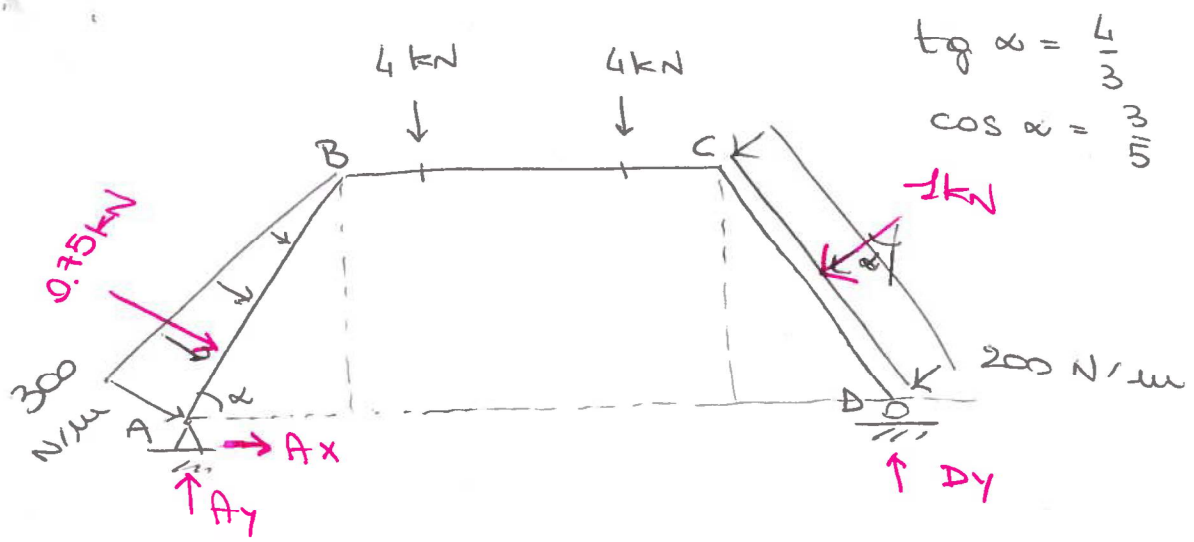


(M)

$\frac{480 \times 6}{2} = 1440$ kft







$\operatorname{tg}\alpha = \frac{4}{3}$

$\cos\alpha = \frac{3}{5}$ $\quad \sin\alpha = \frac{4}{5}$

$\Sigma M_A = 0$ $\quad +0.75 \times \frac{5}{3} + 4 \times 5 + 4 \times 8 + (1\,\text{kN} \cos\alpha) \times 11.5 +$

$- (1\,\text{kN} \sin\alpha) \times 2 - D_y \times 13 = 0$

$D_y = \dfrac{1.25 + 20 + 32 + \frac{34.5}{5} - \frac{8}{5}}{13} = 4.50\ \text{kN}$

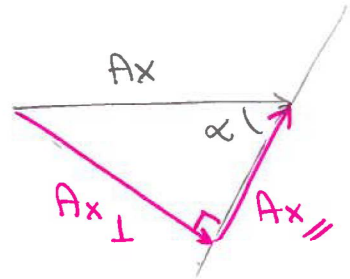
$\Sigma F_y = 0$ $\quad A_y + D_y - 0.75\cos\alpha - 4 - 4 - 1\cos\alpha = 0$

$A_y = 0.75 \times \frac{3}{5} + 8 + \frac{3}{5} - 4.50 = 4.55\ \text{kN}$

$\Sigma F_x = 0$ $\quad +A_x + 0.75 \sin\alpha - 1 \sin\alpha = 0$

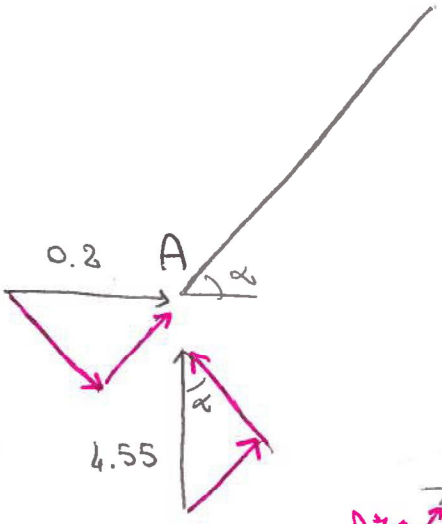
$A_x = 0.25 \times \frac{4}{5} = 0.2\ \text{kN}$

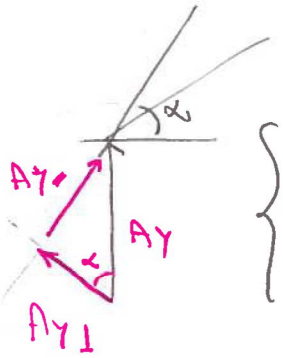
SEGMENT AB



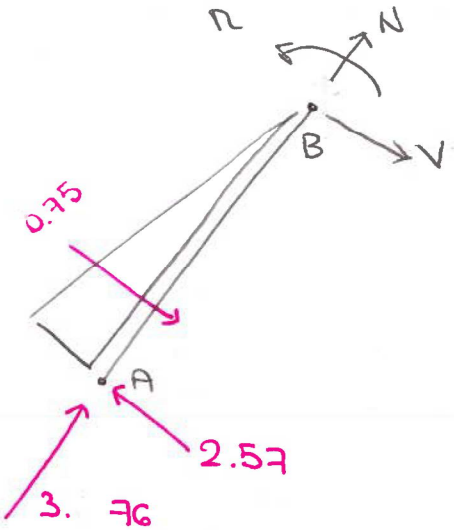
PROJECT REACTION IN // & ⊥ DIRECTIONS:

$$\begin{cases} Ax_{\perp} = Ax \sin\alpha = 0.16\ kN \\ Ax_{//} = Ax \cos\alpha = 0.12\ kN \end{cases}$$





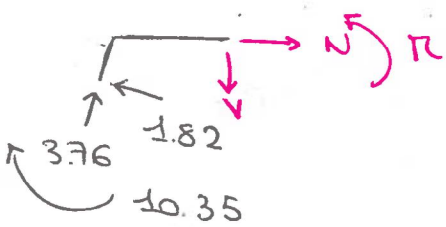
$$\begin{cases} Ay_{//} = Ay \sin\alpha = 3.64\ kN \\ Ay_{\perp} = Ay \cos\alpha = 2.73\ kN \end{cases}$$



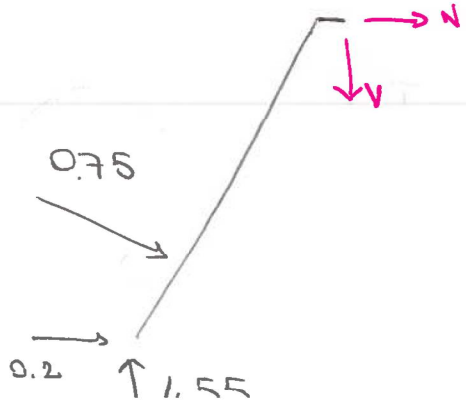
In B:

$$\begin{cases} N = -3.76\ kN \\ V = 2.57 - 0.75 = 1.82\ kN \\ M = 2.57 \times 5 - 0.75 \times \left(2 \times \frac{5}{3}\right) = 10.35\ kNm \end{cases}$$

NODE B

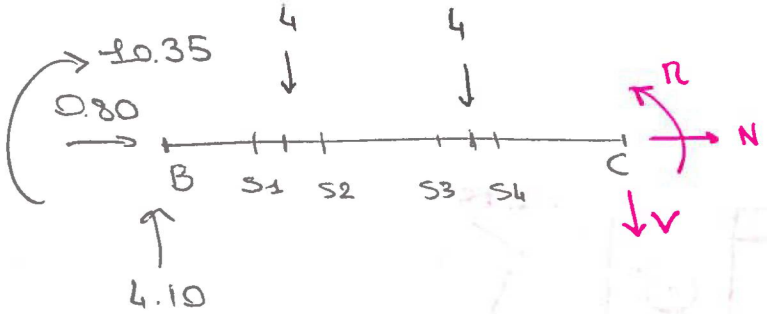


OR



$$\begin{cases} M = 10.35\ kN \\ N = -0.2 - 0.75 \sin\alpha = -0.8\ kN \\ V = 4.55 - 0.75 \cos\alpha = 4.10\ kN \end{cases}$$

SEGMENT BC



S1

$$\begin{cases} N = -0.80\,kN \\ V = 4.10 \\ M = 10.35 + 4.10 \times 2 = 18.55 \end{cases}$$

S2

$$\begin{cases} N = -0.80 \\ V = +0.10\,kN \\ M = 18.55 \end{cases}$$

S3

$$\begin{cases} N = -0.80 \\ V = +0.10\,kN \\ M = 10.35 + 4.10 \times 5 - 4 \times 3 = 18.85 \end{cases}$$

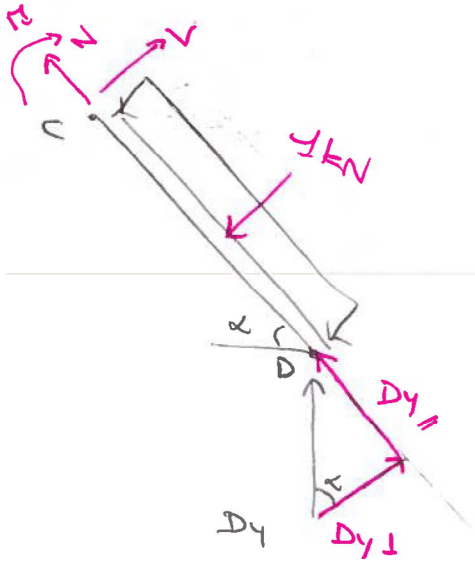
S4

$$\begin{cases} N = -0.80 \\ V = -3.90 \\ M = 18.85 \end{cases}$$

In C

$$\begin{cases} N = -0.80\,kN \\ V = -3.90\,kN \\ M = 10.35 + 4.10 \times 7 - 4 \times 5 - 4 \times 2 = 11.05\,kN\,m \end{cases}$$

SEGMENT CD

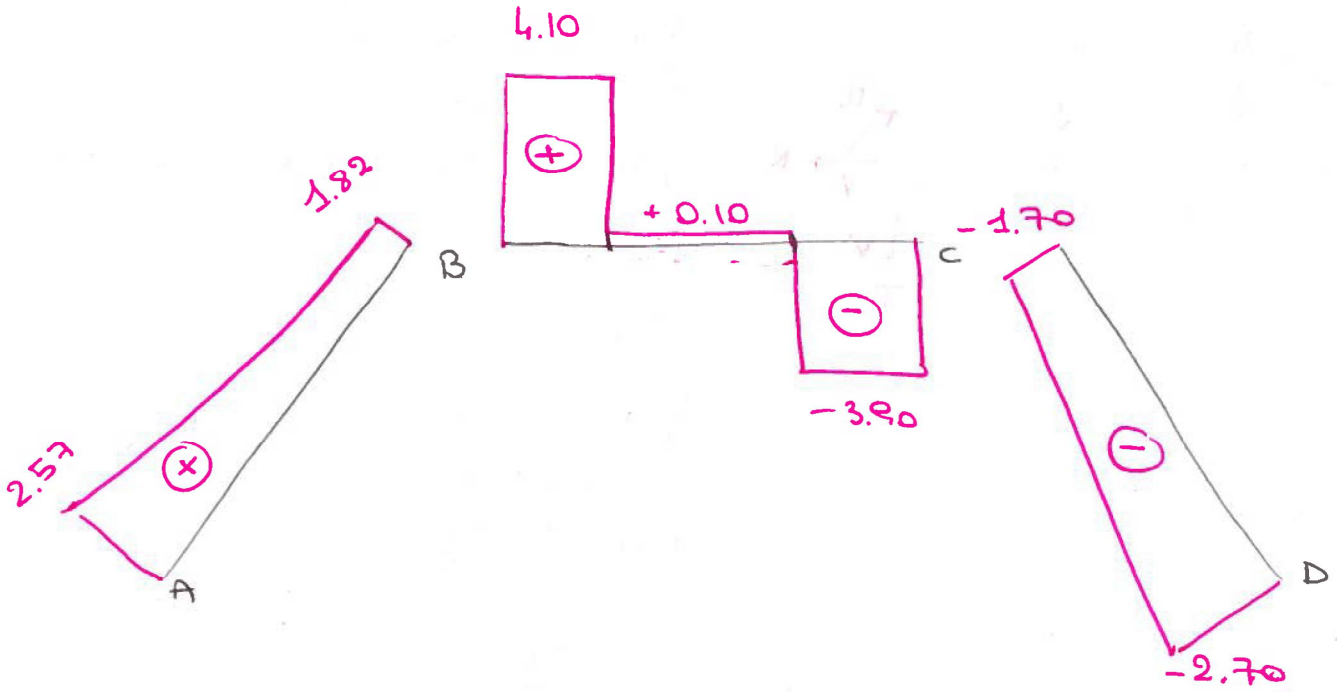


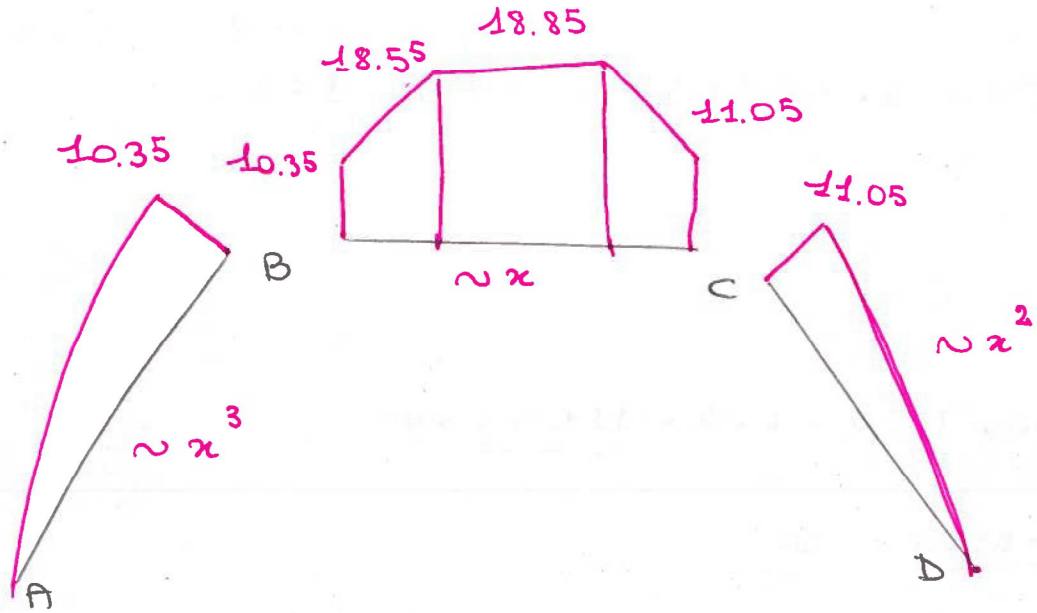
$$D_{y\parallel} = D_y \sin\alpha = 3.6\,kN$$

$$D_{y\perp} = D_y \cos\alpha = 2.7\,kN$$

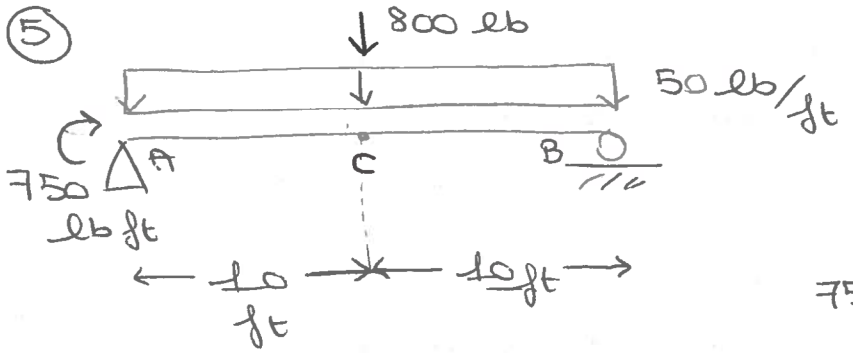
In C

$$\begin{cases} N = -D_{y\parallel} = -3.6\,kN \\ V = 1 - D_{y\perp} = -1.7\,kN \\ M = 2.7 \times 5 - 1 \times 2.5 = 11\,kN\,m \end{cases}$$


4.10
1.82
+0.10
-1.70
B
C
-3.80
2.57
A
D
-2.70


18.85
18.55
11.05
10.35
10.35
11.05
B
C
~x
~x²
~x³
A
D

⑤



① $\Sigma F_x = 0$

$A_x = 0$

② $\Sigma M_A = 0$ (+↺)

$750 + 50 \times 20 \times 10 + 800 \times 10 - B_y \times 20 = 0$

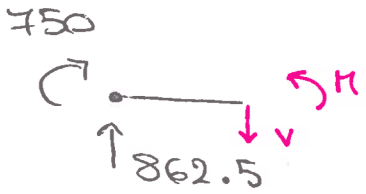
$B_y = 937.5$ lb

③ $\Sigma F_y = 0$

$A_y + B_y - 800 - 50 \times 20 = 0$

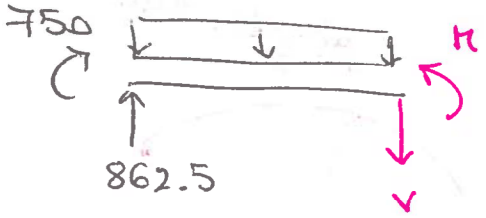
$A_y = 862.5$ lb

NODE A



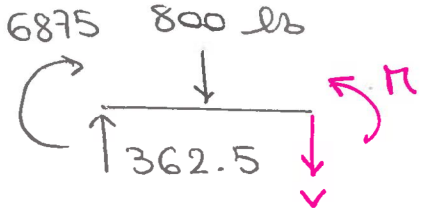
$\begin{cases} V = +862.5 \text{ lb} \\ M = 750 \text{ lb ft} \end{cases}$

AC



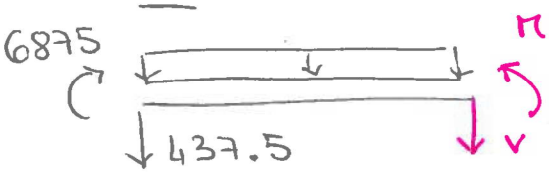
$\begin{cases} V = 862.5 - 50 \times 10 = 362.5 \text{ lb} \\ M = 750 + 862.5 \times 10 - 50 \times \frac{10^2}{2} = 6875 \text{ lb ft} \end{cases}$

NODE C



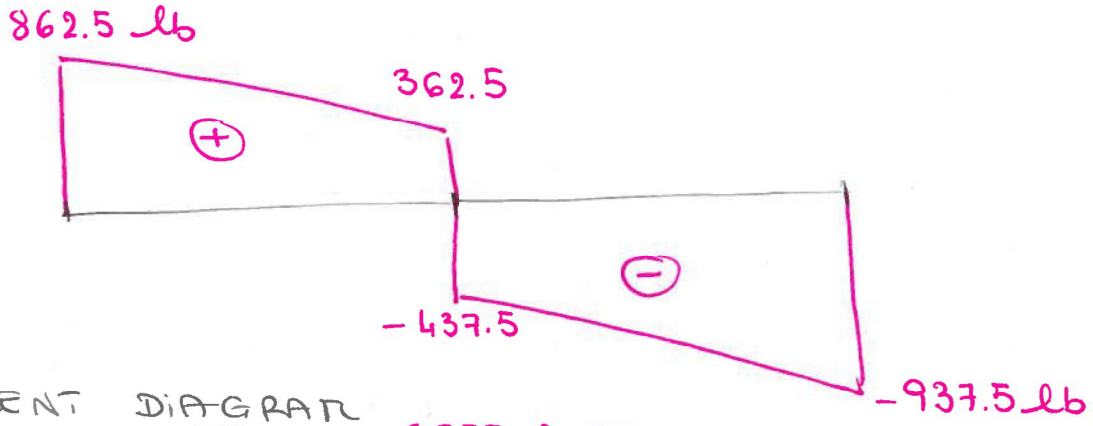
$\begin{cases} V = -437.5 \text{ lb} \\ M = 6875 \text{ lb ft} \end{cases}$

BC

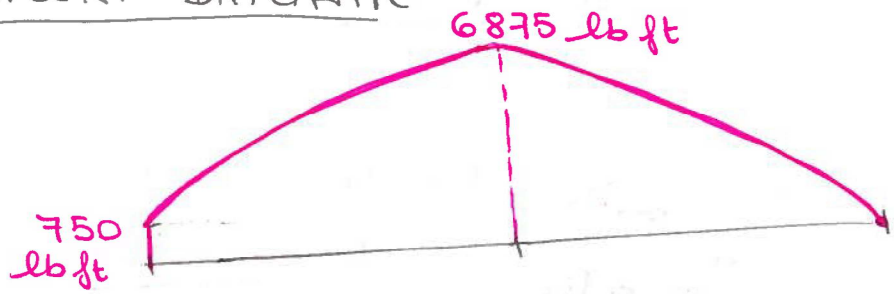


$\begin{cases} V = -437.5 - 50 \times 10 = -937.5 \text{ lb} \\ M = 0 \end{cases}$

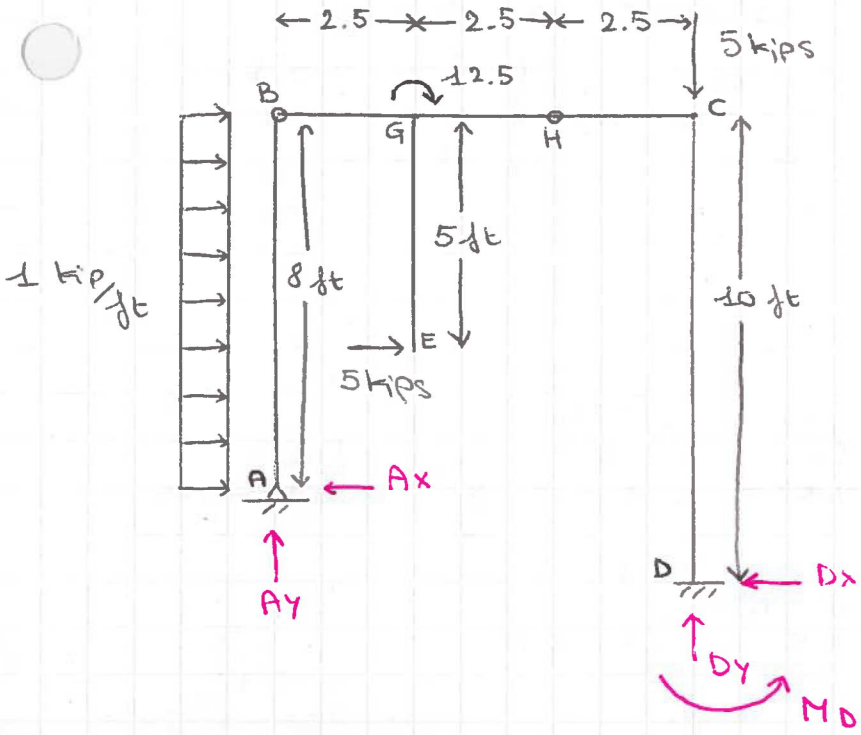
SHEAR DIAGRAM



MOMENT DIAGRAM



PROBLEM 6



$R = 9$
$N = 3$ → $\begin{cases} R = 3N \\ \text{STABLE} \end{cases}$

↓

STATICALLY DETERMINATE

(a) FIND REACTIONS: 5 UNKNOWNS ⇒ 5 EQUILIBRIUM EQUATIONS

(1) $\sum M_B\ (I) = 0$

$A_x \cdot 8 - 1 \cdot 8 \cdot 4 = 0$ → $A_x = 4$ kips

(2) $\sum F_x = 0$

$1 \cdot 8 + 5 - A_x - D_x = 0$ → $D_x = 9$ kips

(3) $\sum M_H\ (I) = 0$

$A_x \cdot 8 + A_y \cdot 5 + 12.5 - 1 \cdot 8 \cdot 4 - 5 \cdot 5 = 0$ → $A_y = 2.5$ kips

(4) $\sum F_y = 0$

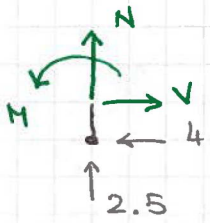
$A_y + D_y - 5 = 0$ → $D_y = 2.5$ kips

(5) $\sum M_H\ (II) = 0$

$5 \cdot 2.5 + D_x \cdot 10 - D_y \cdot 2.5 - M_D = 0$ → $M_D = 96.25$ kip ft

(b) Find N, V, M at beginning and end of each segment

NODE A (IMAGINE TO CUT JUST ABOVE THE NODE)

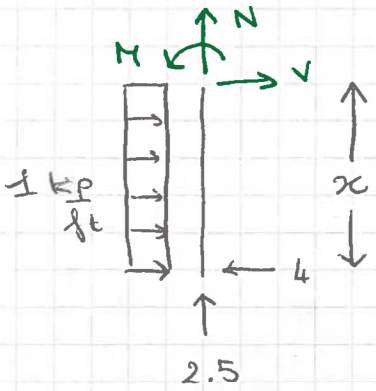


$\Sigma F_x = 0 \qquad V - 4 = 0$

$\Sigma F_y = 0 \qquad N + 2.5 = 0 \quad \Rightarrow \quad \begin{cases} N = -2.5 \text{ kips} \\ V = 4 \text{ kips} \\ M = 0 \end{cases}$

$\Sigma M = 0 \qquad M = 0$

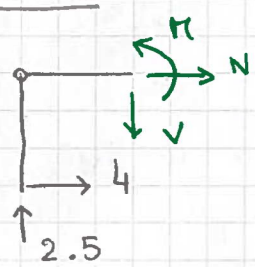
SEGMENT AB



at x: $\begin{cases} N(x) = -2.5 \text{ kips} \\ V(x) = 4 - 1 \cdot x \\ M(x) = 4 \cdot x - 1 \cdot \frac{x^2}{2} \end{cases}$ LINEAR QUADRATIC

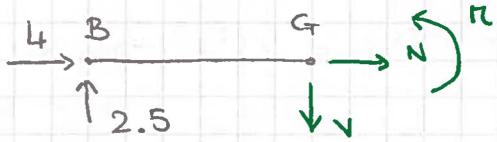
in B: $\begin{cases} N = -2.5 \text{ kips} \\ V = -4 \text{ kips} \\ M = 0 \end{cases}$ (HINGE)

NODE B



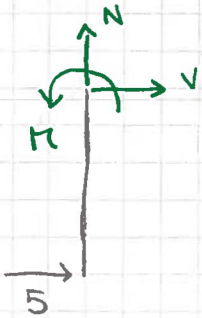
$\begin{cases} N = -4 \text{ kips} \\ V = 2.5 \text{ kips} \\ M = 0 \end{cases}$

SEGMENT BG



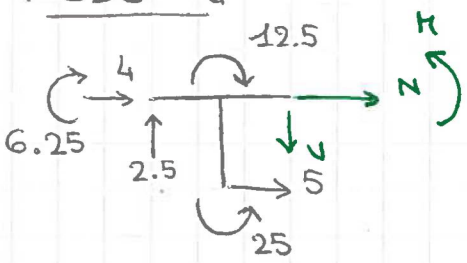
$\begin{cases} N = -4 \text{ kips} \\ V = 2.5 \text{ kips} \\ M = 6.25 \text{ kips ft} \end{cases}$

SEGMENT EG



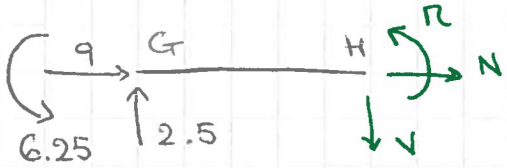
$\begin{cases} N = 0 \\ V = -5 \text{ kips} \\ M = -25 \text{ kip ft} \end{cases}$

NODE G



$$\begin{cases} N = -9 \text{ kips} \\ V = 2.5 \text{ kips} \\ M = -6.25 \text{ kip ft} \end{cases}$$

SEGMENT GH

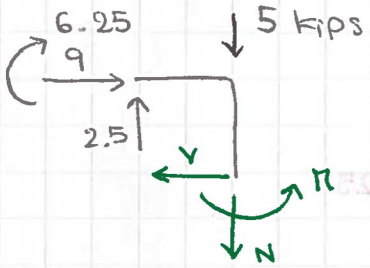


$$\begin{cases} N = -9 \text{ kips} \\ V = 2.5 \text{ kips} \\ M = 0 \quad \text{(HINGE)} \end{cases}$$

SEGMENT HC

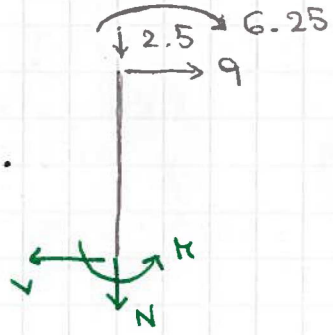
$$\begin{cases} N = -9 \text{ kips} \\ V = 2.5 \text{ kips} \\ M = 6.25 \text{ kip ft} \end{cases}$$

NODE C



$$\begin{cases} N = -2.5 \text{ kips} \\ V = 9 \text{ kips} \\ M = 6.25 \text{ kip ft} \end{cases}$$

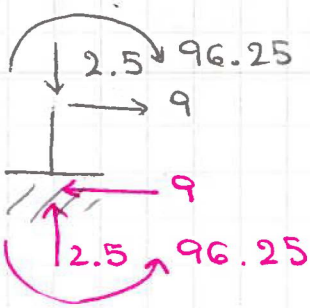
SEGMENT CD



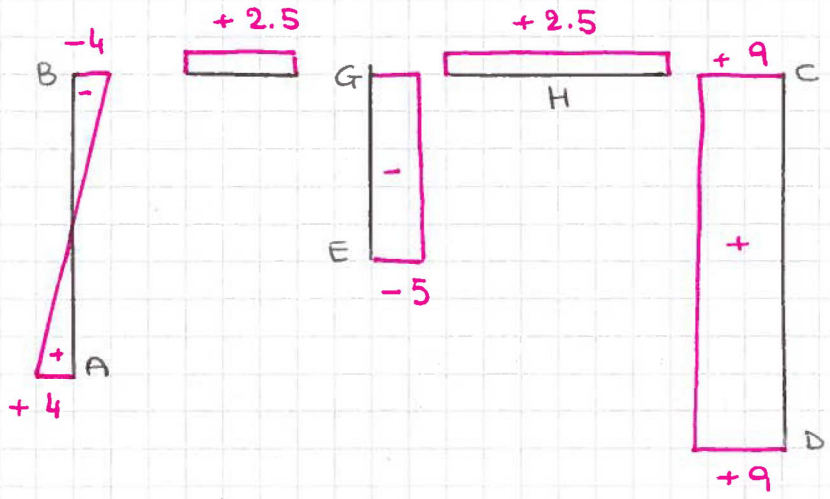
$$\begin{cases} N = -2.5 \text{ kips} \\ V = 9 \text{ kips} \\ M = 96.25 \text{ kip ft} \end{cases}$$

NODE D (CHECK):

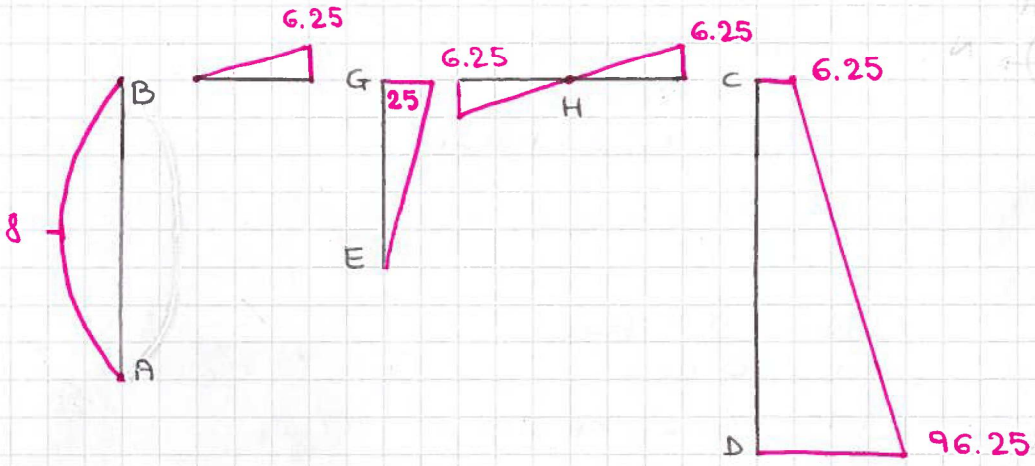
EQUILIBRIUM? YES!



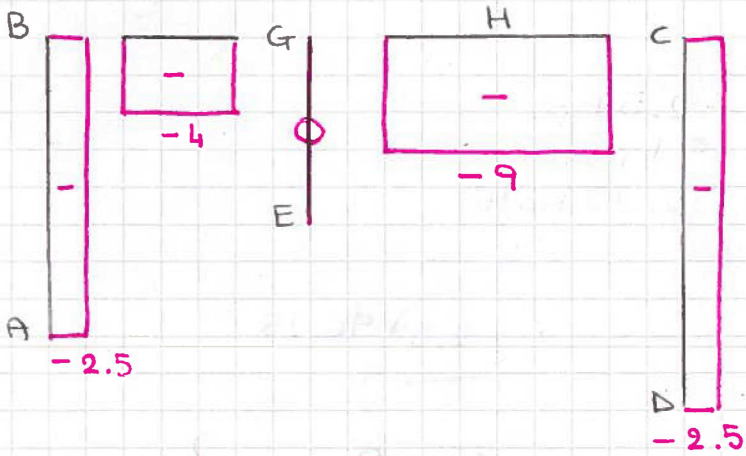
SHEAR DIAGRAM



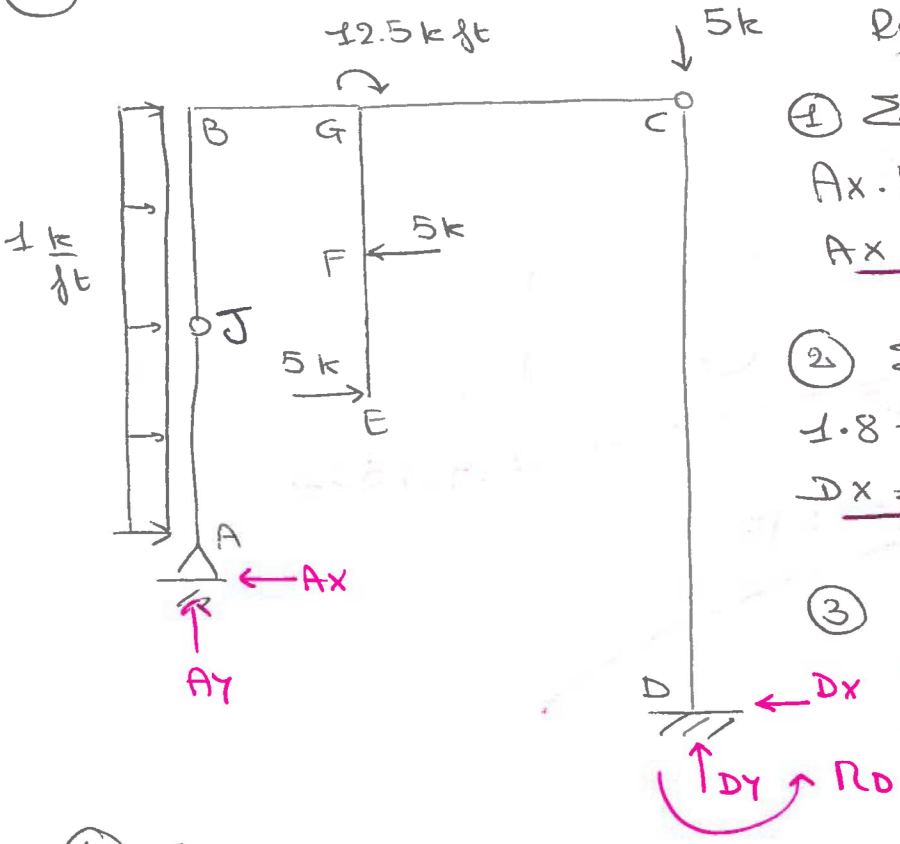
MOMENT DIAGRAM



AXIAL FORCE DIAGRAM



(4)



REACTIONS:

① $\Sigma M_J (I) = 0$

$Ax \cdot 4 - 1 \cdot 4 \cdot 2 = 0$

$Ax = 2$ kips

② $\Sigma Fx = 0$

$1 \cdot 8 + 5 - 5 - Ax - Dx = 0$

$Dx = 6$ kips

③ $\Sigma M_C (II) = 0$

$Dx \cdot 10 - M_D = 0$

$M_D = 60$ k ft

④ $\Sigma M_C (I) = 0$

$Ax \cdot 8 + Ay \times 7.5 + 12.5 + 5 \times 2.5 - 5 \times 5 - 1 \times 8 \times 4 = 0$

$Ay = \dfrac{25 + 32 - 12.5 - 12.5 - 16}{7.5} = 2.13$ k

⑤ $\Sigma Fy = 0$

$Ay + Dy - 5 = 0$ $\quad Dy = 2.87$ k

CHECK: $\Sigma M_G \overset{?}{=} 0$

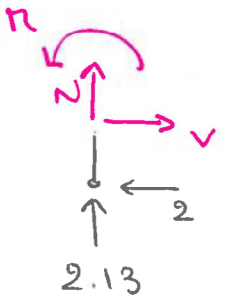
$Ax \cdot 8 + Ay \times 2.5 - 1 \times 8 \times 4 + 12.5 + 5 \times 2.5 - 5 \times 5 + 5 \times 5 +$

$+ Dx \cdot 10 - Dy \cdot 5 - M_D =$

$= 16 + 5.33 - 32 + 12.5 + 12.5 - 25 + 25 + 60 - 14.33 - 60 =$

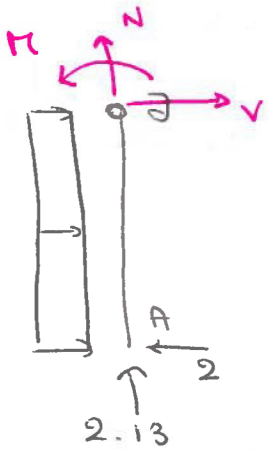
$= 0$ ok REACTIONS ARE CORRECT

NODE A



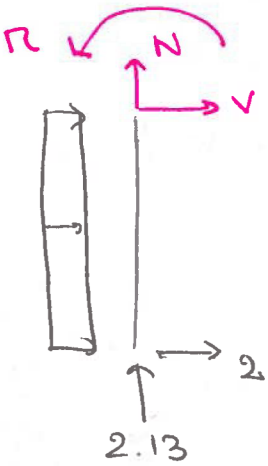
$$\begin{cases} N = -2.13\ k \\ V = 2\ k \\ M = 0 \end{cases}$$

SEGMENT AJ



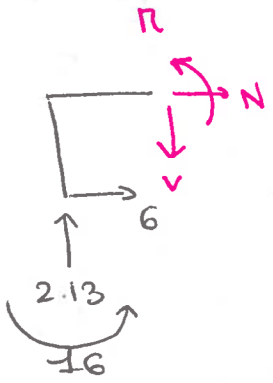
$$\begin{cases} N = -2.13\ k \\ V = -2\ k \\ M = 0 \end{cases}$$

SEGMENT JB



$$\begin{cases} N = -2.13\ k \\ V = -6\ k \\ M = -8-8 = -16\ k\,ft \end{cases}$$

NODE B

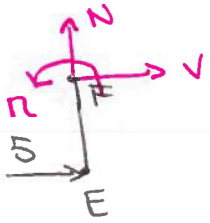


$$\begin{cases} N = -6\ k \\ V = 2.13\ k \\ M = -16\ k\,ft \end{cases}$$

SEGMENT BG

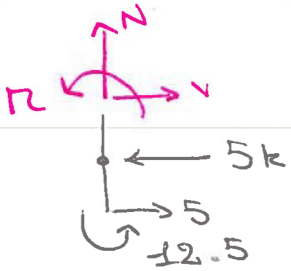
$$\begin{cases} N = -6\ k \\ V = 2.13\ k \\ M = -10.66\ k\,ft \end{cases}$$

SEGMENT EF



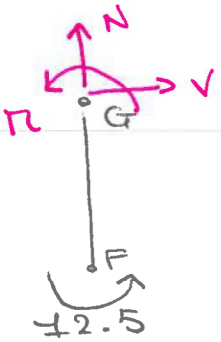
$$\begin{cases} N = 0 \\ V = -5\ k \\ M = -12.5\ k\,ft \end{cases}$$

NODE F



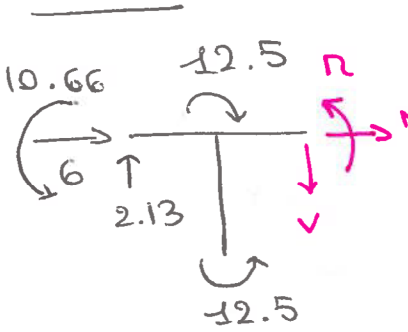
$$\begin{cases} N = 0 \\ V = 0 \\ M = -12.5\ k\,ft \end{cases}$$

SEGMENT FG



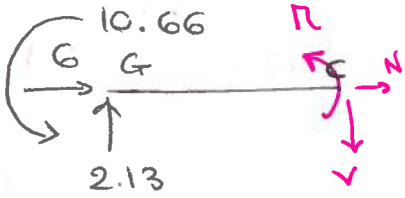
$$\begin{cases} N = 0 \\ V = 0 \\ M = -12.5\ k\,ft \end{cases}$$

NODE G



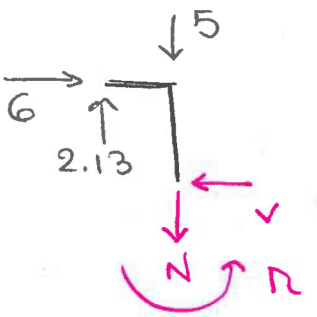
$$\begin{cases} N = -6\,k \\ V = 2.13\,k \\ M = -10.66\,k\,ft \end{cases}$$

SEGMENT GC



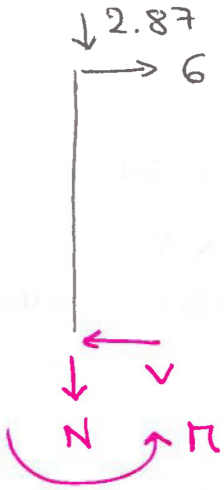
$$\begin{cases} N = -6\,k \\ V = 2.13\,k \\ M = 0 \end{cases}$$

NODE C



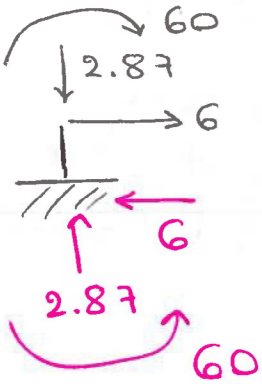
$$\begin{cases} N = -2.87\,k \\ V = 6\,k \\ M = 0 \end{cases}$$

SEGMENT CD



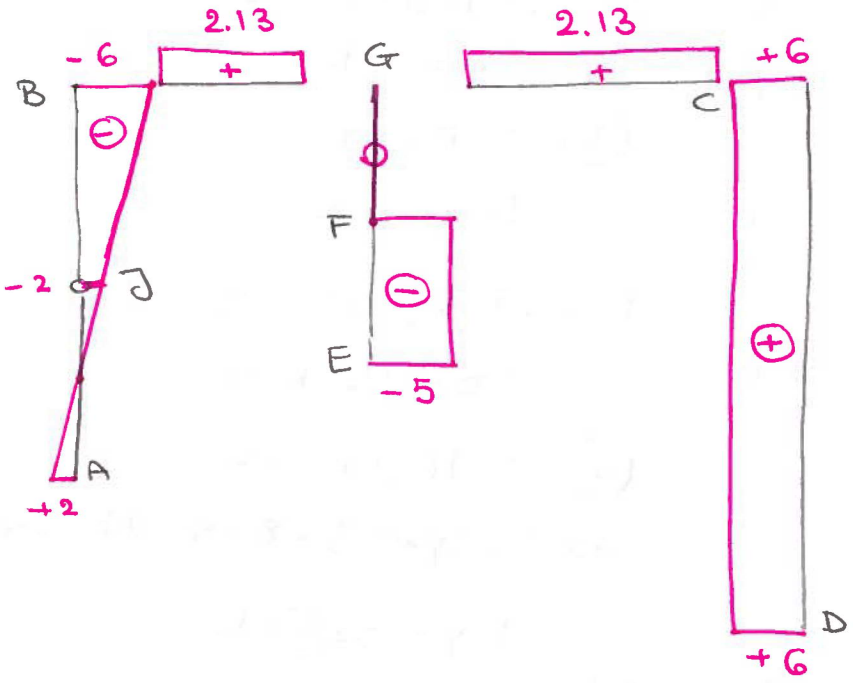
$$\begin{cases} N = -2.87\,k \\ V = 6\,k \\ M = 60\,k\,ft \end{cases}$$

CHECK EQUILIBRIUM OF NODE D:

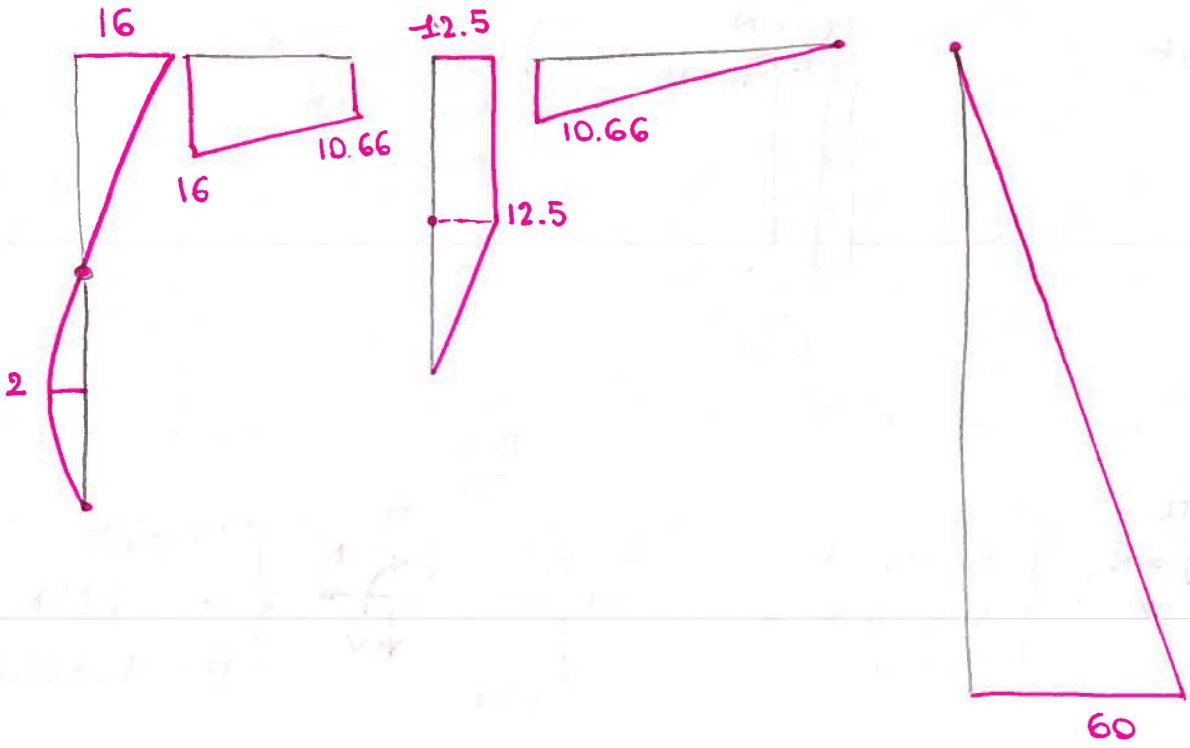


OK!

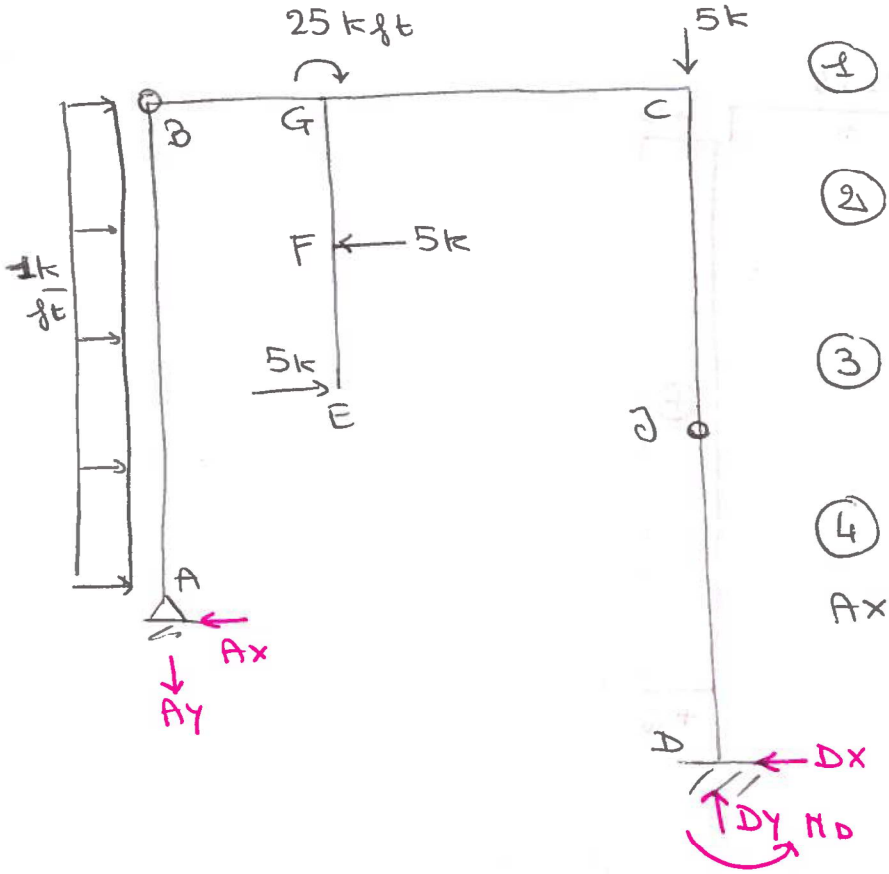
SHEAR DIAGRAM



MOMENT DIAGRAM



(8)



REACTIONS

(1) $\Sigma M_B(I) = 0$

$Ax = 4\ k$

(2) $\Sigma F_x = 0$

$Dx = 4\ k$

(3) $\Sigma M_J(II) = 0$

$M_D = 20\ k\,ft$

(4) $\Sigma M_J(I) = 0$

$Ax \cdot 3 - Ay \cdot 7.5 + 8 \times 1 + 25 - 5 \times 2.5 = 0$

$Ay = 4.33\ k$

(5) $\Sigma F_y = 0$

$Dy = 9.33\ k$

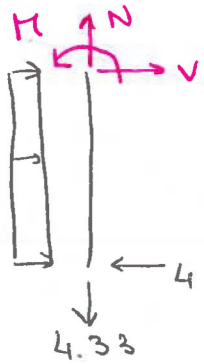
NODE A

$N = +4.33\ k$

$V = 4\ k$

$M = 0$

AB

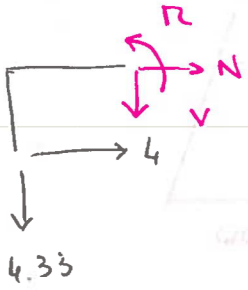


$N = 4.33\ k$

$V = -4\ k$

$M = 0$

NODE B

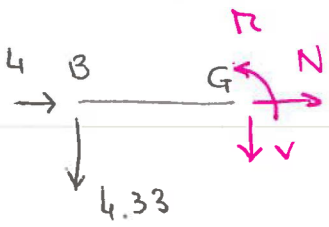


$N = -4\ k$

$V = -4.33\ k$

$M = 0$

BG

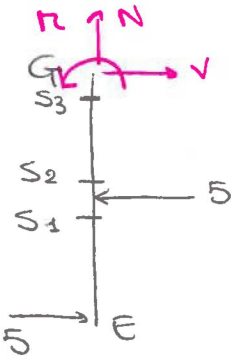


$N = -4\ k$

$V = -4.33\ k$

$M = -10.83\ k\,ft$

EF & FG



S1:

$$\begin{cases} N = 0 \\ V = -5k \\ M = -12.5 k\,ft \end{cases}$$

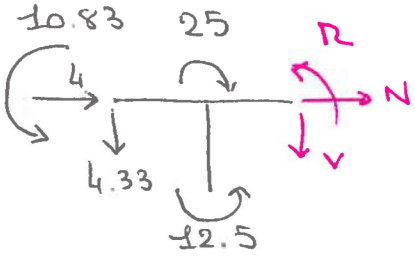
S2:

$$\begin{cases} N = 0 \\ V = 0 \\ M = -12.5 k\,ft \end{cases}$$

S3:

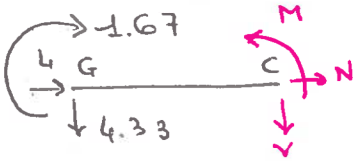
$$\begin{cases} N = 0 \\ V = 0 \\ M = -12.5 k\,ft \end{cases}$$

NODE G



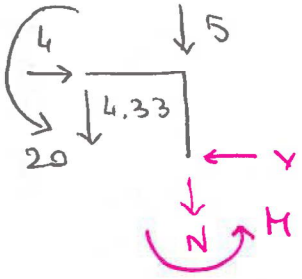
$$\begin{cases} N = 4 k \\ V = -4.33 k \\ M = 1.67 k\,ft \end{cases}$$

GC



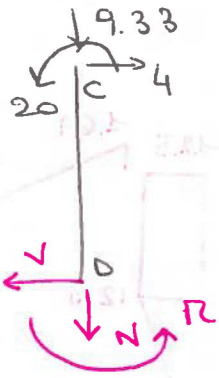
$$\begin{cases} N = -4 k \\ V = -4.33 k \\ M = -20 k\,ft \end{cases}$$

NODE C



$$\begin{cases} N = -9.33 k \\ V = 4 k \\ M = -20 k\,ft \end{cases}$$

CD



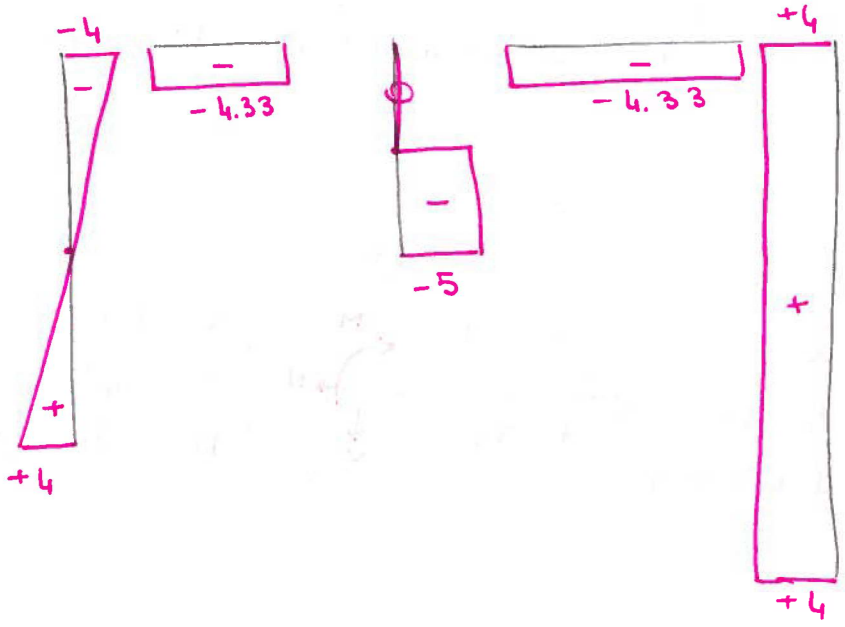
in J:

$$\begin{cases} N = -9.33 k \\ V = 4 k \\ M = 0 \end{cases}$$

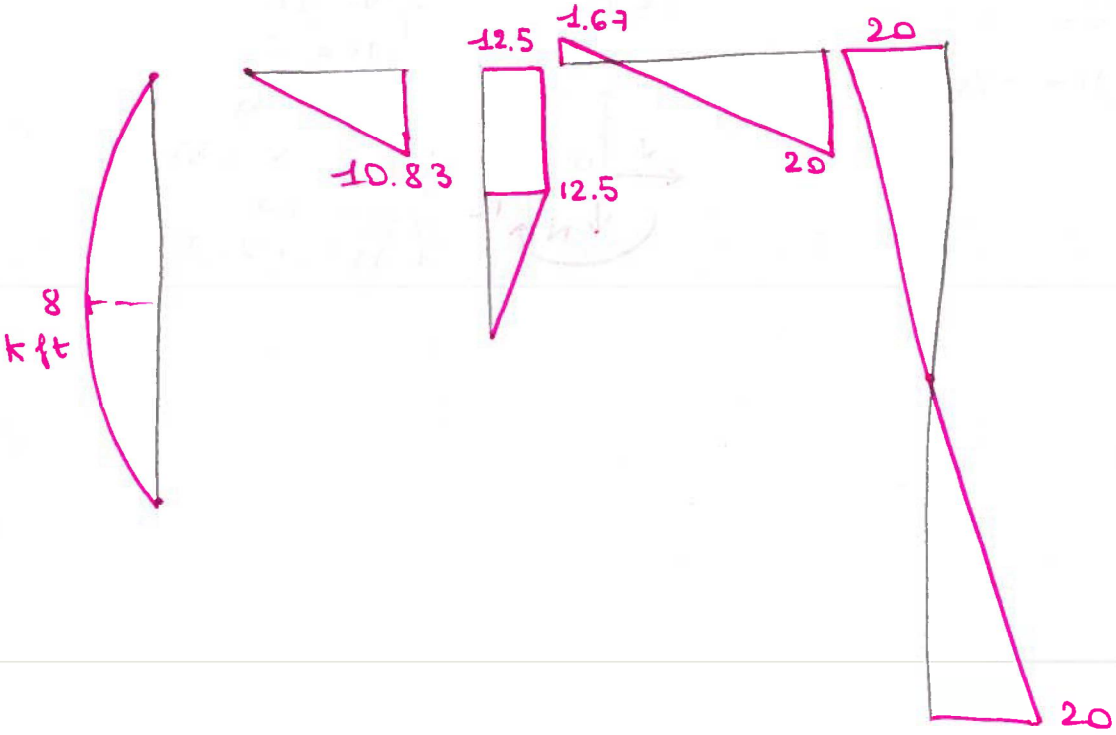
in D:

$$\begin{cases} N = -9.33 k \\ V = 4 k \\ M = +20 k\,ft \end{cases}$$

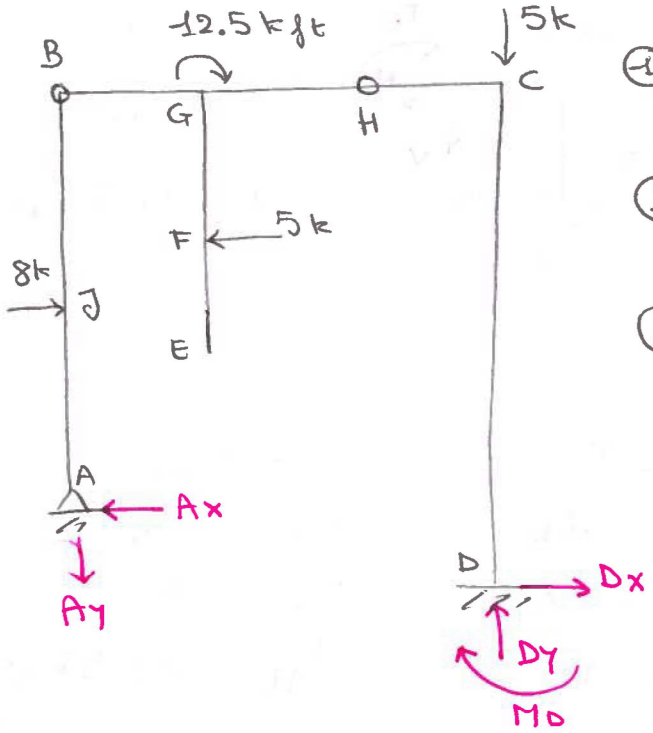
SHEAR DIAGRAM



MOMENT DIAGRAM



(9)



REACTIONS:

① $\Sigma M_B(I) = 0$

$A_x = 4\,k$

② $\Sigma F_x = 0$

$D_x = 1\,k$

③ $\Sigma M_H(I) = 0$

$A_x \cdot 8 - A_y \cdot 5 - 8 \cdot 4 + 12.5 + 5 \times 2.5 = 0$

$A_y = 5\,k$

④ $\Sigma F_y = 0$

$D_y = 10\,k$

⑤ $\Sigma M_H(II) = 0$

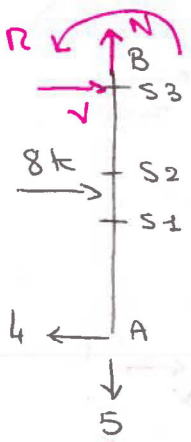
$5 \times 2.5 - D_x \cdot 10 + M_D - D_y \times 2.5 = 0$

$M_D = 22.5\ k\,ft$

NODE A

$\begin{cases} N = +5\,k \\ V = +4\,k \\ M = 0 \end{cases}$

AB



S1:

$\begin{cases} N = 5\,k \\ V = 4\,k \\ M = 16\,k\,ft \end{cases}$

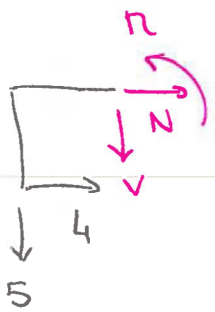
S2:

$\begin{cases} N = 5\,k \\ V = -4\,k \\ M = 16\,k\,ft \end{cases}$

S3:

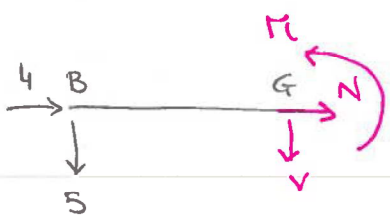
$\begin{cases} N = 5\,k \\ V = -4\,k \\ M = 0 \end{cases}$

NODE B



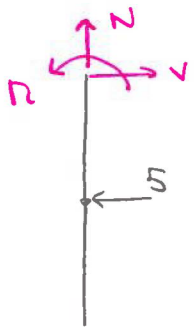
$\begin{cases} N = -4\,k \\ V = -5\,k \\ M = 0 \end{cases}$

BG



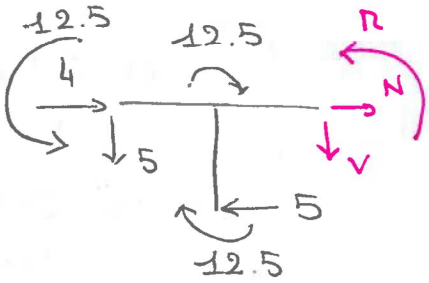
$\begin{cases} N = -4\,k \\ V = -5\,k \\ M = -12.5\,k\,ft \end{cases}$

EG



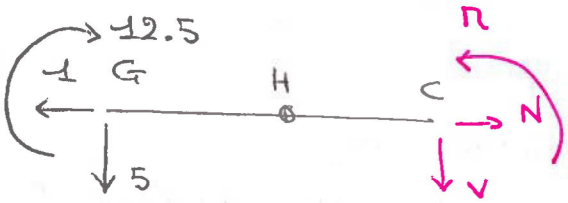
$$\begin{cases} N = 0 \\ V = 5\,k \\ M = 12.5\,k\,ft \end{cases}$$

NODE G



$$\begin{cases} N = 1\,k \\ V = -5\,k \\ M = 12.5\,k\,ft \end{cases}$$

GC



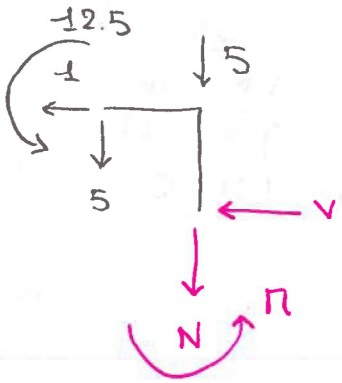
in H:

$$\begin{cases} N = 1\,k \\ V = -5\,k \\ M = 0 \end{cases}$$

in C:

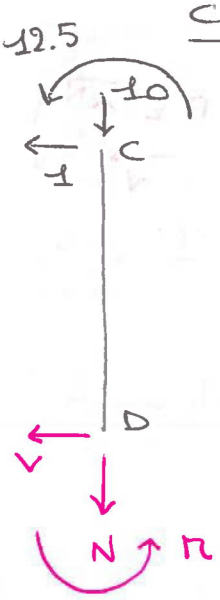
$$\begin{cases} N = 1\,k \\ V = -5\,k \\ M = -12.5\,k\,ft \end{cases}$$

NODE C



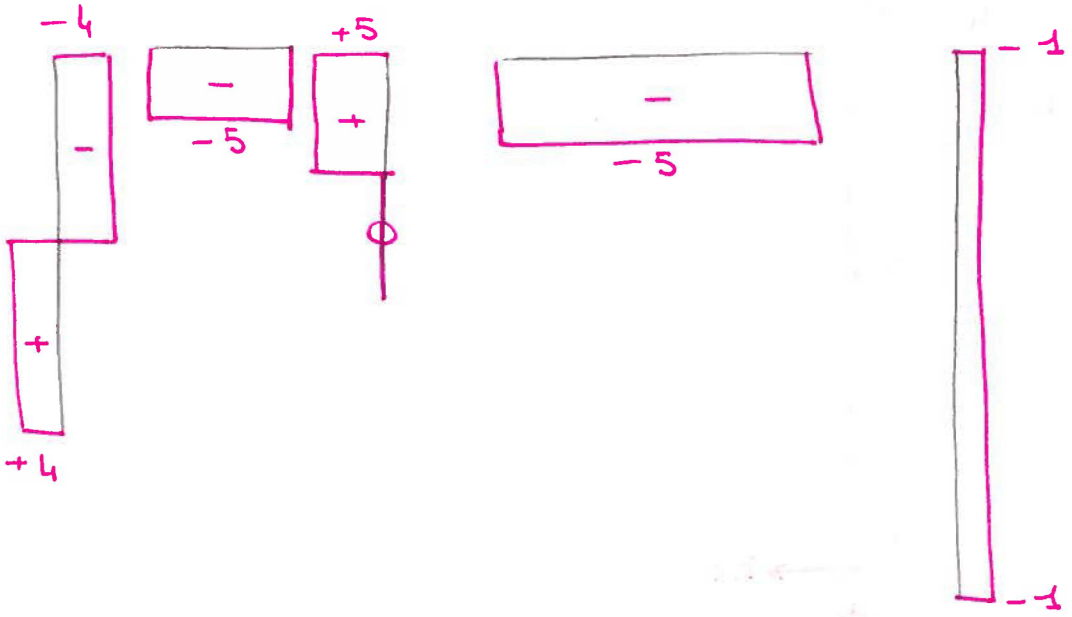
$$\begin{cases} N = -10\,k \\ V = -1\,k \\ M = -12.5\,k\,ft \end{cases}$$

CD

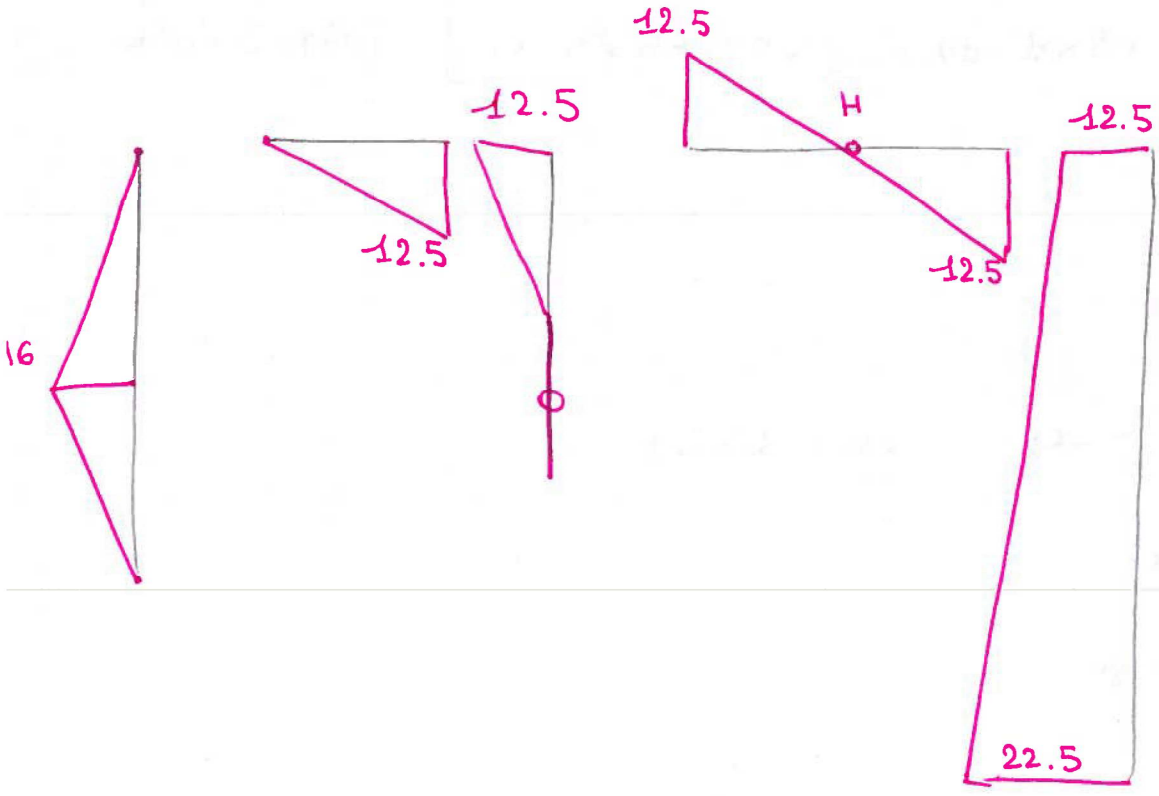


$$\begin{cases} N = -10\,k \\ V = -1\,k \\ M = -22.5\,k\,ft \end{cases}$$

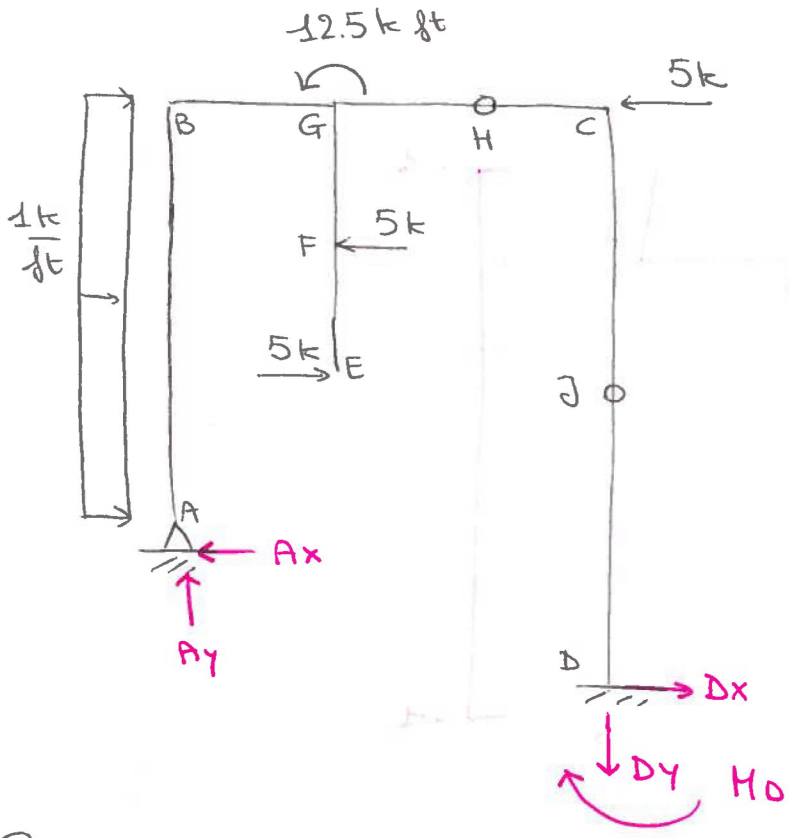
SHEAR DIAGRAM



MOMENT DIAGRAM



(10)



(1) $\Sigma M_H (I) = 0$

$Ax \cdot 8 + Ay \times 5 - 8 \times 4 - 12.5 + 5 \times 2.5 - 5 \times 5 = 0$

(2) $\Sigma M_J (I) = 0$

$Ax \cdot 3 + Ay \times 7.5 + 8 \times 1 - 12.5 - 5 \times 2.5 - 5 \times 5 = 0$

$\Rightarrow \begin{cases} Ax = 4.83k \\ Ay = 3.67k \end{cases}$

(3) $\Sigma F_y = 0$

$Dy = 3.67k$

(4) $\Sigma F_x = 0$

$-Ax + Dx + 8 - 5 = 0 \qquad Dx = 1.83k$

(5) $\Sigma M_J (II) = 0$

$M_D = 9.1 \ k\,ft$

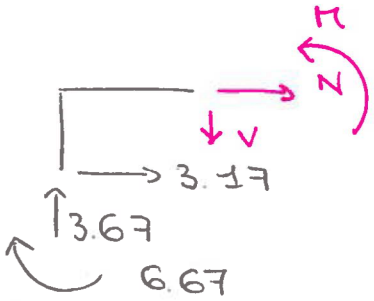
NODE A

$$\begin{cases} N = -3.67\,k \\ V = 4.83\,k \\ M = 0 \end{cases}$$

AB

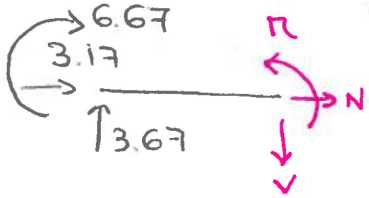
$$\begin{cases} N = -3.67\,k \\ V = -3.17\,k \\ M = 6.67\,k\,ft \end{cases}$$

NODE B



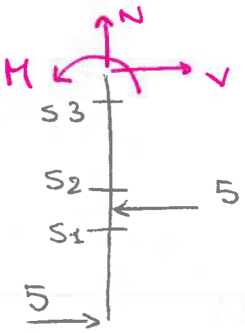
$$\begin{cases} N = -3.17\,k \\ V = 3.67\,k \\ M = 6.67\,k\,ft \end{cases}$$

BG



$$\begin{cases} N = -3.17\,k \\ V = 3.67\,k \\ M = 15.83\,k\,ft \end{cases}$$

EG



S1:

$$\begin{cases} N = 0 \\ V = -5\,k \\ M = -12.5\,k\,ft \end{cases}$$

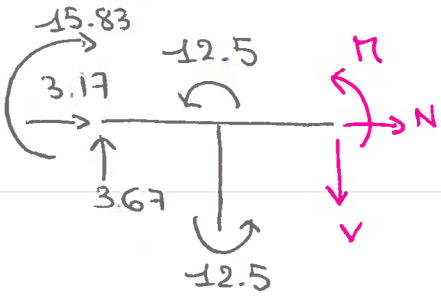
S2:

$$\begin{cases} N = 0 \\ V = 0 \\ M = -12.5\,k\,ft \end{cases}$$

S3:

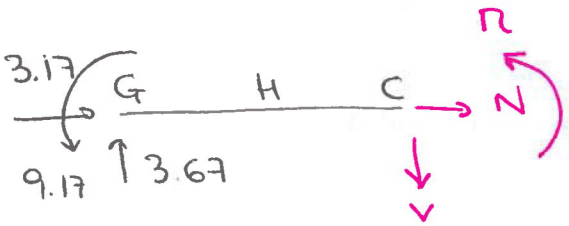
$$\begin{cases} N = 0 \\ V = 0 \\ M = -12.5\,k\,ft \end{cases}$$

NODE G



$$\begin{cases} N = -3.17\,k \\ V = 3.67\,k \\ M = -9.17\,k\,ft \end{cases}$$

GC



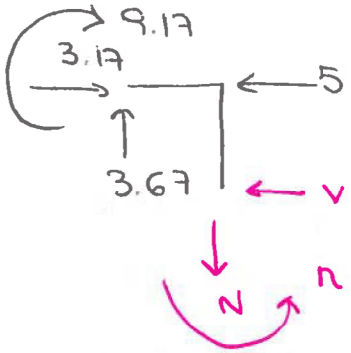
in H:

$$\begin{cases} N = -3.17\,k \\ V = 3.67\,k \\ M = 0 \end{cases}$$

in C:

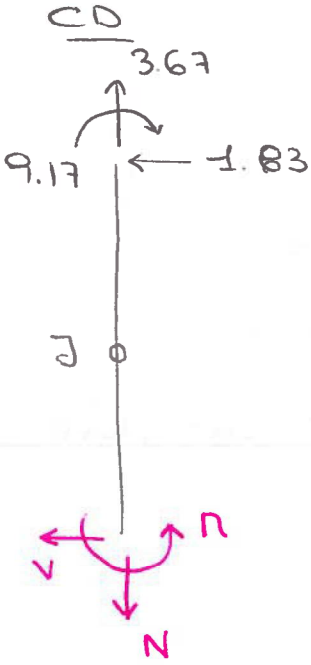
$$\begin{cases} N = -3.17\,k \\ V = 3.67\,k \\ M = 9.17\,k\,ft \end{cases}$$

NODE C



$$\begin{cases} N = 3.67\,k \\ V = -1.83\,k \\ M = 9.17\,k\,ft \end{cases}$$

CD



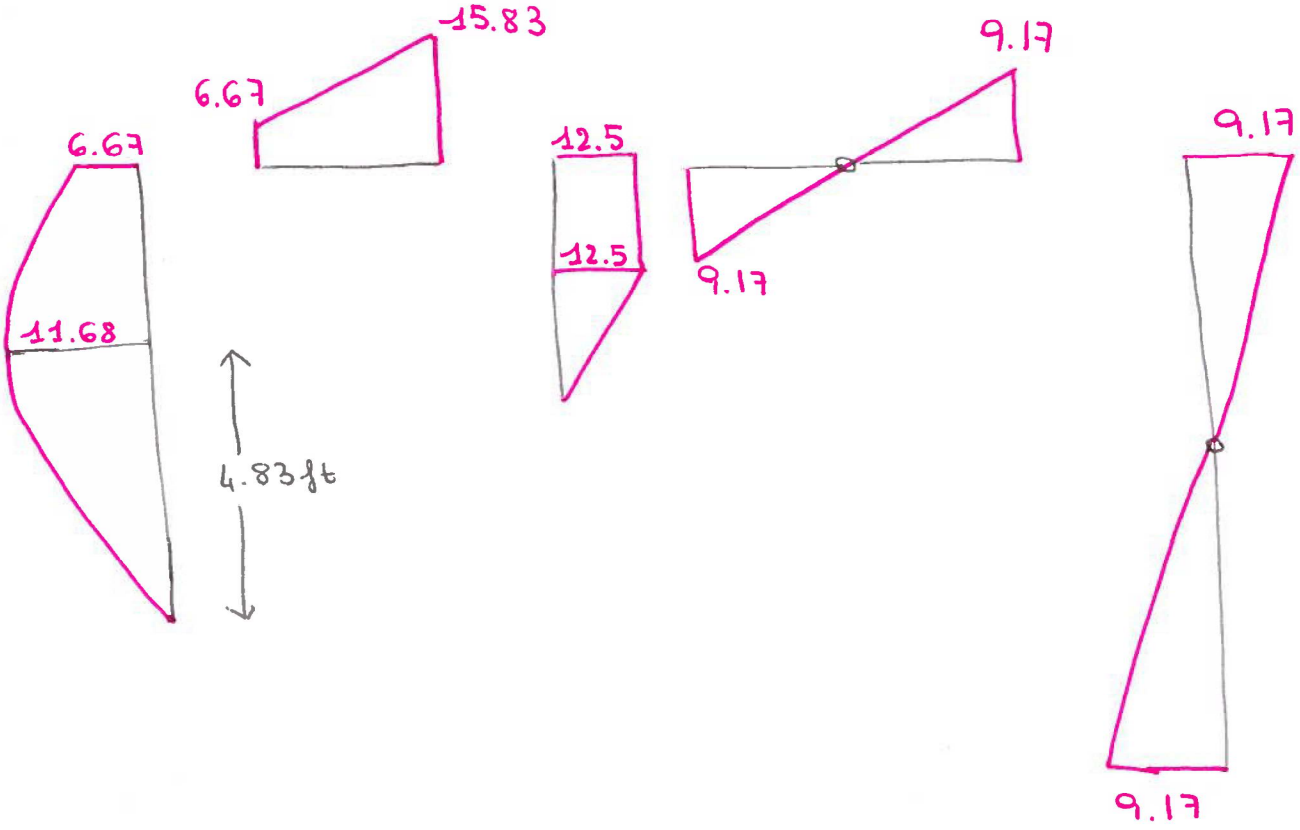
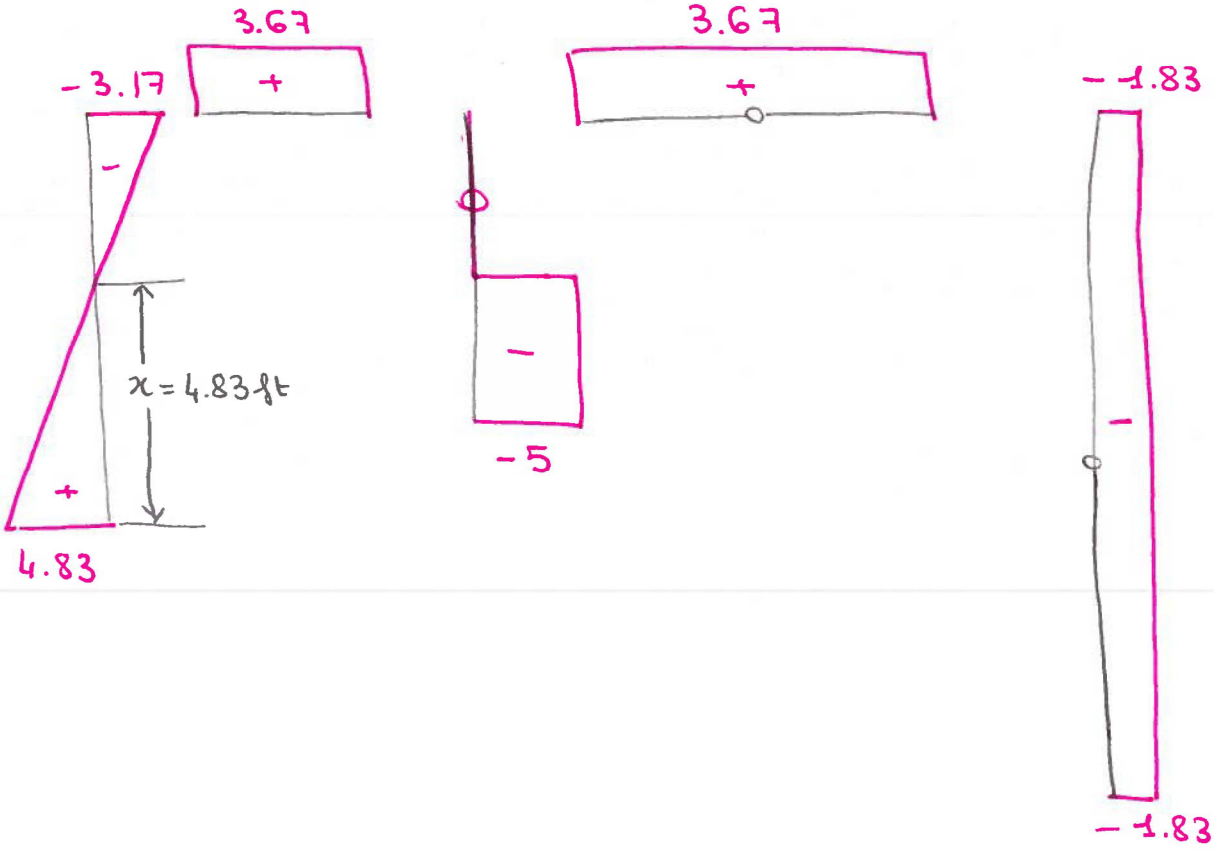
in J:

$$\begin{cases} N = 3.67\,k \\ V = -1.83\,k \\ M = 0 \end{cases}$$

in D:

$$\begin{cases} N = 3.67\,k \\ V = -1.83\,k \\ M = -9.17\,k\,ft \end{cases}$$

MOMENT DIAGRAM



SHEAR DIAGRAM